Name KEY

Math 254, Quiz #12, Summer 2012

Instructions: Show all work. Use exact answers.

1. Integrate.
   a. $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F}(x,y,z) = (2x-y)\hat{\imath} + (3y-z)\hat{\jmath} + (5z-x)\hat{k}$ on the path $\vec{r}(t) = (2t-1)\hat{\imath} + t\hat{\jmath} - t^2\hat{k}$ on $0 \le t \le 1$.

$2\hat{\imath} + 1\hat{\jmath} - 2t\hat{k}$

$$\int_0^1 \langle 4t-2-t,\ 3t+t^2,\ -5t^2-2t+1\rangle \cdot \langle 2,\ +1,\ -2t\rangle\, dt =$$

$3t-2$

$$\int_0^1 6t-4+3t+t^2+10t^3+4t^2-2t\ dt$$

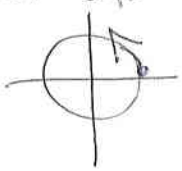
$$\int_0^1 10t^3+5t^2+7t-4\ dt$$

$$\frac{10}{4}t^4+\frac{5}{3}t^3+\frac{7}{2}t^2-4t\Big|_0^1 = \frac{5}{2}+\frac{5}{3}+\frac{7}{2}-4 = 11/3$$

$\oint_C M\,dx + N\,dy$

b. $\int_C (x^2-y)dx + (3xy)dy$ on the closed curve following the circle $x^2+y^2=16$ starting at (4,0). [Hint: use Green's Theorem.]

$$= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$



$\frac{\partial N}{\partial x} = 3y \qquad \frac{\partial M}{\partial y} = -1$

$3y = 3r\sin\theta$

$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 3y-(-1) = 3y+1$

$$\int_0^{2\pi}\int_0^4 (3r\sin\theta+1)\,r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^4 3r^2\sin\theta + r\,dr\,d\theta$$

$$\int_0^{2\pi} r^3\sin\theta + \frac{r^2}{2}\Big|_0^4\, d\theta =$$

$$\int_0^{2\pi} 64\sin\theta + 8\,d\theta$$

$$-64\cos\theta + 8\theta\Big|_0^{2\pi} = -64(1) + 16\pi - (-64(1)) =$$

$$\boxed{16\pi}$$