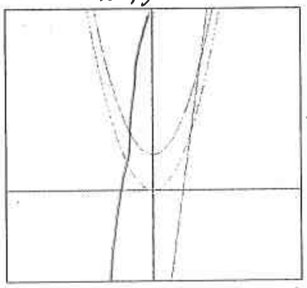
Name KEY

Math 254, Quiz #11, Summer 2012

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the volume of the region bounded by graphs of $z = \sqrt[3]{x^2 - y}e^{12x-y}, y = x^2 + 4, y = x^2, y = 12x - 24, y = 12x + 24$. The region in the plane is shown.



$x^2 - y = -4$ $\quad -4 \le u \le 0$

$x^2 - y = 0$ $\quad -24 \le v \le 24$

$12x - y = 24$ $\quad \displaystyle\int_{-4}^{0}\int_{-24}^{24} u^{1/3} e^{v} \frac{\partial(x,y)}{\partial(u,v)}\, du\, dv$

$12x - y = -24$

$x^2 - y = u$

$-12x + y = -v$

$x^2 - 12x = u - v$

$x^2 - 12x + 36 = u - v + 36$

$\sqrt{(x-6)^2} = \pm\sqrt{u-v+36}$

$\therefore\ x = 6 \pm \sqrt{u-v+36} = 6 \pm (u-v+36)^{1/2}$

$y = 12[6 \pm \sqrt{u-v+36}] - v = 72 \pm 6(u-v+36)^{1/2} - v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \pm\frac{1}{2}(u-v+36)^{-1/2} & \pm\frac{1}{2}(u-v+36)^{-1/2}\cdot(-1) \\ \pm 3(u-v-36)^{-1/2} & \pm 3(u-v+36)^{-1/2}(-1) - 1 \end{vmatrix}$$

$\cancel{\mp\frac{3}{2}(u-v+36)^{-1}} \mp \frac{1}{2}(u-v+36)^{-1/2} - \cancel{\mp\frac{3}{2}(u-v+36)^{-1}} =$

$\mp\frac{1}{2}(u-v+36)^{-1/2}$

$$\boxed{\pm\frac{1}{2}\int_{-4}^{0}\int_{-24}^{24} u^{1/3} e^{v} (u-v+36)^{-1/2}\, du\, dv}$$

2. Find the curl and divergence of the vector field $\vec{F}(x,y,z) = (2xz + 2xy^2)\hat{\imath} + (2x^2y + z^3)\hat{\jmath} + (x^2 + 3yz^2)\hat{k}$. Use that information to determine if the field is conservative. If it is, also find the potential function.

$\nabla \times F =$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xz + 2xy^2 & 2x^2y + z^3 & x^2 + 3yz^2 \end{vmatrix} = \underbrace{(3z^2 - 3z^2)}_{=0}\hat{\imath} - \underbrace{(2x - 2x)}_{=0}\hat{\jmath} + \underbrace{(4x - 4x)}_{=0}\hat{k} = \boxed{\vec{0}}$$

$\nabla \cdot F = 2z + 2y^2 + 2x^2 + 6z = \boxed{2x^2 + 2y^2 + 8z}$

$\nabla \times F = 0$ so field is conservative

$\int 2xz + 2xy^2\, dx = x^2z + x^2y^2 + g(y,z)$

$\int 2x^2y + z^3\, dy = x^2y^2 + z^3y + h(x,z)$

$\int x^2 + 3yz^2\, dz = x^2z + yz^3 + k(x,y)$

$$\boxed{f(x,y,z) = x^2z + x^2y^2 + yz^3 + C}$$