# Lagrange Multipliers Key Math 254 (Summer 2012)

1. $w = x^2 - y^2$ s.t. $x + 2y = 5$

$F(x,y,\lambda) = x^2 - y^2 - \lambda x - 2\lambda y + 5\lambda$

$F_x = 2x - \lambda = 0 \Rightarrow \lambda = 2x$

$F_y = -2y - 2\lambda = 0 \Rightarrow \lambda = y \Rightarrow 2x = y$

$F_\lambda$: $x + 2y - 5 = 0$

$x + 2(2x) - 5 = 0$

$5x - 5 = 0$

$x = 1 \Rightarrow 2(1) = y = 2$ $\quad (1,2)$

2. $w = x^2 - 10x + y^2 - 14y + 28$ s.t. $x + y = 10$

$F(x,y,\lambda) = x^2 - 10x + y^2 - 14y + 28 - \lambda x - \lambda y + 10\lambda$

$F_x = 2x - 10 - \lambda = 0 \Rightarrow \lambda = 2x - 10$

$F_y = 2y - 14 - \lambda = 0 \quad \lambda = 2y - 14$

$F_\lambda$: $x + y = 10$ $\quad 2x - 10 = 2y - 14$

$x - 5 = y - 7$

$+5 \quad +5$

$x = y - 2$

$y - 2 + y = 10$

$2y = 12$

$y = 6$

$x = 6 - 2 = 4$ $\quad (4,6)$

3. $w = xy + yz + xz$ s.t. $x + y + z = 1$, $x - 2y + 3z = 15$

$F(x,y,z,\lambda,\mu) = xy + yz + xz - \lambda x - \lambda y - \lambda z + \lambda - \mu x + 2\mu y - 3\mu z + 15\mu$

$F_x = y + z - \lambda - \mu = 0 \quad \lambda = y + z - \mu$

$F_y = x + z - \lambda + 2\mu = 0 \quad \lambda = x + z + 2\mu$

$F_z = y + x - \lambda - 3\mu = 0 \quad \lambda = y + x - 3\mu$

$y + \cancel{z} - \mu = 2\mu + x + \cancel{z} \quad 3\mu = y - x$

$\cancel{x} + z + 2\mu = y + \cancel{x} - 3\mu \quad \mu = \frac{1}{3}(y - x)$

$z - y = -5\mu \quad \mu = -\frac{1}{5}(z - y)$

$\frac{1}{3}(y - x) = -\frac{1}{5}(z - y) * 15$

$5y - 5x = 3y - 3z \Rightarrow 2y = 5x - 3z$

#3 continued

$F_\lambda$: $x+y+z=1$ $\quad x+\frac{1}{2}(5x-3z)=1$

$F_\mu$: $x-2y+3z=15$ $\quad x-(5x-3z)+3z=15$

$2y=5x-3z$

$x+\frac{5}{2}x-\frac{3}{2}z=1 \Rightarrow 7x-3z=1$ ×2

$x-5x+3z+3z=15 \Rightarrow -4x+6z=15$

$14x-6z=2$

$-4x+6z=15$

$10x=17$

$x=17/10$

$\frac{7(\frac{17}{10})-1}{3}=z=\frac{109}{30}$

$y=\frac{1}{2}(5\cdot\frac{17}{10}-3\cdot\frac{109}{30})=-6/5$

$(17/10, -6/5, 109/30)$ *

4. $w=x^2+y^2+z^2$ s.t. $x+2z=6$, $x+y=12$

$F(x,y,z,\lambda,\mu)=x^2+y^2+z^2-\lambda x-2\lambda z+6\lambda-\mu x-\mu y+12\mu$

$f_x=2x-\lambda-\mu=0 \Rightarrow 2x-2y-z=0$

$F_y=2y-\mu=0 \Rightarrow \mu=2y$

$f_z=2z-2\lambda=0 \Rightarrow \lambda=z$

$F_\lambda$: $x+2z=6$ $\quad z=\frac{6-x}{2}$ $\quad z=0$

$F_\mu$: $x+y=12$ $\quad y=12-x$ $\quad y=12-6=6$

$2x-2y-z=0$

$2x-2(12-x)-\frac{6-x}{2}=0$

$4x-4(12-x)-(6-x)=0$

$4x-48+4x-6+x=0$

$9x-54=0$

$9x=54$

$x=6$

$(6,6,0)$

* The last time I did this problem I got $(27/38, -5/19, 113/38)$
So no promises on which is correct!