Homework #1 Key
Math 266

5. This is pretty straightforward, using the normal words for the logical operators.

a) Sharks have not been spotted near the shore.

b) Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.

c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.

d) If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.

e) If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.

f) If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.

g) Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.

h) Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore. Note that we were able to incorporate the parentheses by using the word "either" in the second half of the sentence.

11. a) "But" is a logical synonym for "and" (although it often suggests that the second part of the sentence is likely to be unexpected). So this is $r \wedge \neg p$.

b) Because of the agreement about precedence, we do not need parentheses in this expression: $\neg p \wedge q \wedge r$.

c) The outermost structure here is the conditional statement, and the conclusion part of the conditional statement is itself a biconditional: $r \rightarrow (q \leftrightarrow \neg p)$.

d) This is similar to part **(b)**: $\neg q \wedge \neg p \wedge r$.

e) This one is a little tricky. The statement that the condition is necessary is a conditional statement in one direction, and the statement that this condition is not sufficient is the negation of the conditional statement in the other direction. Thus we have the structure $(\text{safe} \rightarrow \text{conditions}) \wedge \neg(\text{conditions} \rightarrow \text{safe})$. Fleshing this out gives our answer: $(q \rightarrow (\neg r \wedge \neg p)) \wedge \neg((\neg r \wedge \neg p) \rightarrow q)$. There are some logically equivalent correct answers as well.

f) We just need to remember that "whenever" means "if" in logic: $(p \wedge r) \rightarrow \neg q$.

29. To construct the truth table for a compound proposition, we work from the inside out. In each case, we will show the intermediate steps. In part **(a)**, for example, we first construct the truth table for $p \vee q$, then the truth table for $p \oplus q$, and finally combine them to get the truth table for $(p \vee q) \rightarrow (p \oplus q)$. For parts **(a)**, **(b)**, and **(c)** we have the following table (column five for part **(a)**, column seven for part **(b)**, column eight for part **(c)**).

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$	$p \wedge q$	$(p \oplus q) \rightarrow (p \wedge q)$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	F	F	T	T	F
T	F	T	T	T	F	F	T
F	T	T	T	T	F	F	T
F	F	F	F	T	F	T	F

For part **(d)** we have the following table.

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

For part **(e)** we need eight rows in our truth table, because we have three variables.

p	q	r	$\neg p$	$\neg r$	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg r$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F

For part **(f)** we have the following table.

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

31. The techniques are the same as in Exercises 27–30. For parts **(a)** and **(b)** we have the following table (column four for part **(a)**, column six for part **(b)**).

p	q	$\neg q$	$p \to \neg q$	$\neg p$	$\neg p \leftrightarrow q$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	F

For parts **(c)** and **(d)** we have the following table (columns six and seven, respectively).

p	q	$p \to q$	$\neg p$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$	$(p \to q) \wedge (\neg p \to q)$
T	T	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	T	F	T	F

For parts **(e)** and **(f)** we have the following table (this time we have not explicitly shown the columns for negation). Column five shows the answer for part **(e)**, and column seven shows the answer for part **(f)**.

p	q	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$	$\neg p \leftrightarrow \neg q$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	T	T	F	T	T	T
T	F	F	T	T	F	T
F	T	F	T	T	F	T
F	F	T	F	T	T	T

33. The techniques are the same as in Exercises 27–32, except that there are now three variables and therefore eight rows. For part **(a)**, we have

p	q	r	$\neg q$	$\neg q \vee r$	$p \to (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

For part (b), we have

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Parts (c) and (d) we can combine into a single table.

p	q	r	$p \to q$	$\neg p$	$\neg p \to r$	$(p \to q) \vee (\neg p \to r)$	$(p \to q) \wedge (\neg p \to r)$
T	T	T	T	F	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	F

For part (e) we have

p	q	r	$p \leftrightarrow q$	$\neg q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Finally, for part (f) we have

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

58. We can draw no conclusions. A knight will declare himself to be a knight, telling the truth. A knave will lie and assert that he is a knight. Since everyone will say "I am a knight," we can determine nothing.

7. De Morgan's laws tell us that to negate a conjunction we form the disjunction of the negations, and to negate a disjunction we form the conjunction of the negations.

a) This is the conjunction "Jan is rich, and Jan is happy." So the negation is "Jan is not rich, or Jan is not happy."

b) This is the disjunction "Carlos will bicycle tomorrow, or Carlos will run tomorrow." So the negation is "Carlos will not bicycle tomorrow, and Carlos will not run tomorrow." We could also render this as "Carlos will neither bicycle nor run tomorrow."

c) This is the disjunction "Mei walks to class, or Mei takes the bus to class." So the negation is "Mei does not walk to class, and Mei does not take the bus to class." (Maybe she gets a ride with a friend.) We could also render this as "Mei neither walks nor takes the bus to class."

d) This is the conjunction "Ibrahim is smart, and Ibrahim is hard working." So the negation is "Ibrahim is not smart, or Ibrahim is not hard working."

9. We construct a truth table for each conditional statement and note that the relevant column contains only T's. For parts **(a)** and **(b)** we have the following table (column four for part **(a)**, column six for part **(b)**).

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

For parts **(c)** and **(d)** we have the following table (columns five and seven, respectively).

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$	$p \wedge q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	F	T
F	F	T	T	T	F	T

For parts **(e)** and **(f)** we have the following table (this time we have not explicitly shown the columns for negation). Column five shows the answer for part **(e)**, and column seven shows the answer for part **(f)**.

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T	T	F	T	F	T
T	F	F	T	T	T	T
F	T	T	F	T	F	T
F	F	T	F	T	T	T

29. We will show that if $p \rightarrow q$ and $q \rightarrow r$ are both true, then $p \rightarrow r$ is true. Thus we want to show that if p is true, then so is r. Given that p and $p \rightarrow q$ are both true, we conclude that q is true; from that and $q \rightarrow r$ we conclude that r is true, as desired. This can also be done with a truth table.

33. To show that these are *not* logically equivalent, we need only find one assignment of truth values to p, q, r, and s for which the truth values of $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ differ. Let us try to make the first one false. That means we have to make $r \rightarrow s$ false, so we want r to be true and s to be false. If we let p and q be false, then each of the other three simple conditional statements ($p \rightarrow q$, $p \rightarrow r$, and $q \rightarrow s$) will be true. Then $(p \rightarrow q) \rightarrow (r \rightarrow s)$ will be $\mathrm{T} \rightarrow \mathrm{F}$, which is false; but $(p \rightarrow r) \rightarrow (q \rightarrow s)$ will be $\mathrm{T} \rightarrow \mathrm{T}$, which is true.

34. This time the truth table needs $2^4 = 16$ rows.

p	q	r	s	$p \to q$	$(p \to q) \to r$	$((p \to q) \to r) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

7. a) This statement is that for every x, if x is a comedian, then x is funny. In English, this is most simply stated, "Every comedian is funny."

b) This statement is that for every x in the domain (universe of discourse), x is a comedian *and* x is funny. In English, this is most simply stated, "Every person is a funny comedian." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of boring comedians; it's surely false, because there exist lots of x for which $C(x)$ is false. This illustrates the fact that you rarely want to use conjunctions with universal quantifiers.

c) This statement is that there exists an x in the domain such that if x is a comedian then x is funny. In English, this might be rendered, "There exists a person such that if s/he is a comedian, then s/he is funny." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of funny comedians; it's surely true, because there exist lots of x for which $C(x)$ is false (recall the definition of the truth value of $p \to q$). This illustrates the fact that you rarely want to use conditional statements with existential quantifiers.

d) This statement is that there exists an x in the domain such that x is a comedian and x is funny. In English, this might be rendered, "There exists a funny comedian" or "Some comedians are funny" or "Some funny people are comedians."

9. a) We assume that this sentence is asserting that the same person has both talents. Therefore we can write $\exists x(P(x) \land Q(x))$.

b) Since "but" really means the same thing as "and" logically, this is $\exists x(P(x) \land \neg Q(x))$

c) This time we are making a universal statement: $\forall x(P(x) \lor Q(x))$

d) This sentence is asserting the nonexistence of anyone with either talent, so we could write it as $\neg\exists x(P(x) \lor Q(x))$. Alternatively, we can think of this as asserting that everyone fails to have either of these talents, and we obtain the logically equivalent answer $\forall x \neg(P(x) \lor Q(x))$. Failing to have either talent is equivalent to having neither talent (by De Morgan's law), so we can also write this as $\forall x((\neg P(x)) \land (\neg Q(x))$. Note that it would *not* be correct to write $\forall x((\neg P(x)) \lor (\neg Q(x))$ nor to write $\forall x \neg(P(x) \land Q(x))$.

15. Recall that the integers include the positive and negative integers and 0.

a) This is the well-known true fact that the square of a real number cannot be negative.

b) There are two *real* numbers that satisfy $n^2 = 2$, namely $\pm\sqrt{2}$, but there do not exist any *integers* with this property, so the statement is false.

c) If n is a positive integer, then $n^2 \geq n$ is certainly true; it's also true for $n = 0$; and it's trivially true if n is negative. Therefore the universally quantified statement is true.

d) Squares can never be negative; therefore this statement is false.

21. **a)** One would hope that if we take the domain to be the students in your class, then the statement is true. If we take the domain to be all students in the world, then the statement is clearly false, because some of them are studying only other subjects.

b) If we take the domain to be United States Senators, then the statement is true. If we take the domain to be college football players, then the statement is false, because some of them are younger than 21.

c) If the domain consists of just Princes William and Harry of Great Britain (sons of the late Princess Diana), then the statement is true. It is also true if the domain consists of just one person (everyone has the same mother as him- or herself). If the domain consists of all the grandchildren of Queen Elizabeth II of Great Britain (of whom William and Harry are just two), then the statement is false.

d) If the domain consists of Bill Clinton and George W. Bush, then this statement is true because they do not have the same grandmother. If the domain consists of all residents of the United States, then the statement is false, because there are many instances of siblings and first cousins, who have at least one grandmother in common.

25. Let $P(x)$ be "x is perfect"; let $F(x)$ be "x is your friend"; and let the domain (universe of discourse) be all people.

a) This means that everyone has the property of being not perfect: $\forall x \neg P(x)$. Alternatively, we can write this as $\neg\exists x\, P(x)$, which says that there does not exist a perfect person.

b) This is just the negation of "Everyone is perfect": $\neg\forall x\, P(x)$.

c) If someone is your friend, then that person is perfect: $\forall x(F(x) \rightarrow P(x))$. Note the use of conditional statement with universal quantifiers.

d) We do not have to rule out your having more than one perfect friend. Thus we have simply $\exists x(F(x) \wedge P(x))$. Note the use of conjunction with existential quantifiers.

e) The expression is $\forall x(F(x) \wedge P(x))$. Note that here we did use a conjunction with the universal quantifier, but the sentence is not natural (who could claim this?). We could also have split this up into two quantified statements and written $(\forall x\, F(x)) \wedge (\forall x\, P(x))$.

f) This is a disjunction. The expression is $(\neg\forall x\, F(x)) \vee (\exists x \neg P(x))$.

27. In all of these, we will let $Y(x)$ be the propositional function that x is in your school or class, as appropriate.

a) If we let $V(x)$ be "x has lived in Vietnam," then we have $\exists x\, V(x)$ if the universe is just your schoolmates, or $\exists x(Y(x) \land V(x))$ if the universe is all people. If we let $D(x, y)$ mean that person x has lived in country y, then we can rewrite this last one as $\exists x(Y(x) \land D(x, \text{Vietnam}))$.

b) If we let $H(x)$ be "x can speak Hindi," then we have $\exists x\, \neg H(x)$ if the universe is just your schoolmates, or $\exists x(Y(x) \land \neg H(x))$ if the universe is all people. If we let $S(x, y)$ mean that person x can speak language y, then we can rewrite this last one as $\exists x(Y(x) \land \neg S(x, \text{Hindi}))$.

c) If we let $J(x)$, $P(x)$, and $C(x)$ be the propositional functions asserting x's knowledge of Java, Prolog, and C++, respectively, then we have $\exists x(J(x) \land P(x) \land C(x))$ if the universe is just your schoolmates, or $\exists x(Y(x) \land J(x) \land P(x) \land C(x))$ if the universe is all people. If we let $K(x, y)$ mean that person x knows programming language y, then we can rewrite this last one as $\exists x(Y(x) \land K(x, \text{Java}) \land K(x, \text{Prolog}) \land K(x, \text{C++}))$.

d) If we let $T(x)$ be "x enjoys Thai food," then we have $\forall x\, T(x)$ if the universe is just your classmates, or $\forall x(Y(x) \to T(x))$ if the universe is all people. If we let $E(x, y)$ mean that person x enjoys food of type y, then we can rewrite this last one as $\forall x(Y(x) \to E(x, \text{Thai}))$.

e) If we let $H(x)$ be "x plays hockey," then we have $\exists x\, \neg H(x)$ if the universe is just your classmates, or $\exists x(Y(x) \land \neg H(x))$ if the universe is all people. If we let $P(x, y)$ mean that person x plays game y, then we can rewrite this last one as $\exists x(Y(x) \land \neg P(x, \text{hockey}))$.

35. a) As we saw in Example 13, this is true, so there is no counterexample.

b) Since 0 is neither greater than nor less than 0, this is a counterexample.

c) This proposition says that 1 is the only integer—that every integer equals 1. If is obviously false, and any other integer, such as -111749, provides a counterexample.