MTH 264, Quiz #9, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Determine whether the infinite series converges or diverges. State which test you used. For any power series, say on which interval (if any) the series converges.

   a. $\sum_{n=0}^{\infty}\left(\frac{-2n}{5n+1}\right)^n$ $\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{-2n}{5n+1}\right)\right|^n} = \lim_{n\to\infty}\frac{2n}{5n+1} = \frac{2}{5} < 1$ converges by root test

   b. $\sum_{n=1}^{\infty} n^2\left(\frac{3}{8}\right)^n$ $\lim_{n\to\infty}\frac{(n+1)^2\left(\frac{3}{8}\right)^{n+1}}{n^2\left(\frac{3}{8}\right)^n} = \lim_{n\to\infty}\left(\frac{3}{8}\right)\cdot\cancel{\lim_{n\to\infty}\frac{(n+1)^2}{n^2}}^{\,1} = \frac{3}{8} < 1$

      Converges by ratio test

   c. $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n!}$ $\lim_{n\to\infty}\frac{(x-2)^{n+1}}{(n+1)!}\cdot\frac{n!}{(x-2)^n} = \lim_{n\to\infty}\frac{(x-2)}{n+1} = 0 < 1$

      converges by ratio test for all real #'s

   d. $\sum_{n=0}^{\infty}\frac{11}{7}\left(\frac{x}{4}\right)^n$ $\lim_{n\to\infty}\frac{\frac{11}{7}\left(\frac{x}{4}\right)^{n+1}}{\frac{11}{7}\left(\frac{x}{4}\right)^n} = \lim_{n\to\infty}\frac{x}{4} = \frac{x}{4} < 1$ converges by ratio test on $(-4,4)$

2. Find a power series for the functions centered at c=0.

   a. $f(x) = \frac{12}{3-2x} = \frac{4}{1-\frac{2}{3}x}$ $\sum_{n=0}^{\infty}4\left(\frac{2}{3}x\right)^n$

   b. $f(x) = \frac{5x}{1+x^2}$ $\sum_{n=0}^{\infty}5x(-x^2)^n = \sum_{n=0}^{\infty}5(-1)^n x^{2n+1}$