MTH 264, Quiz #8, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Determine whether the infinite series converges or diverges. If it converges, say what it converges to.

   a. $\sum_{n=0}^{\infty}\left(-\frac{7}{5}\right)^n$ diverges $r = \frac{-7}{5}$ $|r| > 1$

   b. $\sum_{n=1}^{\infty}\frac{2}{n(n+2)}$ $\frac{A}{n} + \frac{B}{n+2} \rightarrow An + 2A + Bn = 2$, $A + B = 0$, $A = 1, B = -1$ telescoping $k=2$ converges

   $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right) = 1 + \frac{1}{2} - \left[\lim_{n\to\infty} \cancel{\frac{1}{n+1} + \frac{1}{n+2}}\right]^{\to 0} = \frac{3}{2}$

2. Determine whether the infinite series converges or diverges. State which test you used. [Note: you may not use the root or ratio tests.]

   a. $\sum_{n=0}^{\infty}\frac{n-2}{n+1}$ diverges nth root test (divergence test)

   b. $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ diverges by integral test

   $\int_1^{\infty}\frac{x}{x^2+1}\,dx = \frac{1}{2}\ln|x^2+1|\Big|_1^{\infty} \to \infty$

   c. $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n+1}}$ Converges $\lim_{n\to\infty}\frac{n^{3/2}}{n\sqrt{n+1}} = 1$ $n^{3/2}$ converges by the p-series test

   Converges by the limit comparison test

   d. $\sum_{n=0}^{\infty}\frac{\cos n\pi}{n^2+1}$ $\cos n\pi = (-1)^n$

   $\lim_{n\to\infty}\frac{1}{n^2+1} = 0$ Converges by alternating series test