MTH 264, Quiz #7, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Simplify $\frac{(3n+1)!}{(3n-1)!}$ $= \frac{(3n+1)(3n)\cancel{(3n-1)!}}{\cancel{(3n-1)!}} = (3n+1)(3n)$

2. Write the first five terms of the sequence. n=0 start

a. $(-1)^{n+1}n2^n$

$-(0)(1)$ $(+1)(1)2$ $(-1)(2)2^2$ $(+1)(3)2^3$ $(-1)(4)2^4$ $(1)(5)2^5$

0, 2, -8, 24, -64, 160

b. $a_1 = 4, a_{k+1} = \frac{1}{2}a_k^2$

4, 8, 32, 512, 131,072

3. Determine if the sequence converges. Is it bounded? Is it monotonic?

a. $a_n = \frac{n^2}{2^{n+2}}$

$\lim_{n\to\infty} \frac{n^2}{2^{n+2}} = 0$ yes, it converges

bounded between 0 and 1

monotonic after $n=3$

b. $a_n = \frac{\sin(n^2)}{n}$

$\lim_{n\to\infty} \frac{\sin(n^2)}{n} = 0$ (by Squeeze Theorem

bounded between -1 and 1

it is not monotonic

$0 \le |\sin(n^2)| \le 1$

$\lim_{n\to\infty} \frac{0}{n} \le \lim \frac{|\sin(n^2)|}{n} \le \lim \frac{1}{n}$

$= 0$