Name _____ KEY _____

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the arc length of the function $y = x^2$ on the interval $[0,1]$.

$f' = 2x$

$$S = \int_0^1 \sqrt{1+4x^2}\, dx \approx 1.4789\ldots$$

2. Find the surface area of the surface generated by revolving the curve $y = \sqrt{x} + 2$ on the interval $[1,9]$ around the y-axis.

$y' = \frac{1}{2\sqrt{x}}$

$$S = 2\pi \int_1^9 x \sqrt{1 + \frac{1}{4x}}\, dx = 2\pi \int_1^9 x \sqrt{\frac{4x+1}{4x}}\, dx =$$

$$2\pi \int_1^9 \frac{x}{2\sqrt{x}} \sqrt{4x+1}\, dx = \cancel{2}\pi \int_1^9 \sqrt{x}\sqrt{4x+1}\, dx = \pi \int_1^9 \sqrt{4x^2+x}\, dx$$

$$\approx \pi\left[81.967\ldots\right] \approx 257.50777\ldots$$

3. A force of 20 N stretches a spring 50 centimeters on a mechanical device for driving fence posts. Find the work done in stretching the spring an additional 70 centimeters. $50\,cm = \frac{1}{2}m$

$F = kx$ $20 = k\left(\frac{1}{2}\right) \rightarrow k = 40$

$$W = \int_{0.5}^{1.2} 40x\, dx = 20x^2 \Big|_{0.5}^{1.2} = 20(1.2)^2 - 20(0.5) =$$

$$23.8$$

4. Find the value of K that makes $f(x) = Kx^3$ a valid probability distribution on the interval $[0,4]$.

$$\int_0^4 Kx^3\, dx = \frac{K}{4}x^4 \Big|_0^4 = 64K = 1$$

$$\Rightarrow K = \frac{1}{64}$$