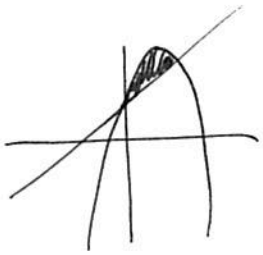
MTH 264, Quiz #1, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the area of the region bounded by $y = -x^2 + 7x + 1, y = 2x + 1$. Sketch the region.



$-x^2+7x+1 = 2x+1$

$-x^2 + 5x = 0$

$-x(x-5) = 0$

$x = 0, x = 5$

$$\int_0^5 -x^2+7x+1-(2x+1)\,dx = \int_0^5 -x^2+5x\,dx =$$

$$-\frac{1}{3}x^3+\frac{5}{2}x^2\Big|_0^5 = -\frac{125}{3}+\frac{5}{2}\cdot 25 = \frac{125}{6}$$

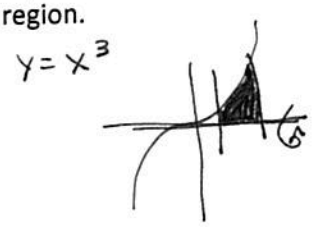
2. Find the volume of the solid of revolution for the region bounded by $y = 16 - x^2, y = 0, x = 2, x = 4$ rotated around the y-axis. Use the shell method. Sketch the region.

$$V = 2\pi\int_2^4 x(16-x^2)\,dx = 2\pi\int_2^4 16x - x^3\,dx =$$

$$2\pi\left[8x^2-\frac{1}{4}x^4\right]_2^4 = 2\pi\left[8\cdot 16 - 64 - 8\cdot 4 + 4\right] =$$

$$2\pi[36] = 72\pi$$

3. Find the volume of the solid of revolution for the region bounded by $x = \sqrt[3]{y}, y = 0, x = 1, x = 2$ rotated around the x-axis. Use the washer or disk method. Sketch the region.



$$\pi\int_1^2 (x^3)^2\,dx = \pi\int_1^2 x^6\,dx =$$

$$\frac{\pi}{7}x^7\Big|_1^2 = \frac{\pi}{7}[128-1] = \frac{127\pi}{7}$$