MTH 264, Quiz #15, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the slope of the tangent line to the graph $r = 2 + 3\cos\theta$ at the point $(-1, \pi)$.

$$\frac{dy}{dx} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta} \qquad r' = -3\sin\theta$$

$$= \frac{(-1)(-1) + 0(0)}{-(-1)(0) + (0)(-1)} = \text{undefined}$$

vertical tangent

2. Find the area of the inner loop to the graph $r = 3 + 6\sin\theta$. Sketch the graph.

$3 + 6\sin\theta = 0$

$-\frac{1}{2} = \sin\theta$

$-\pi/6$

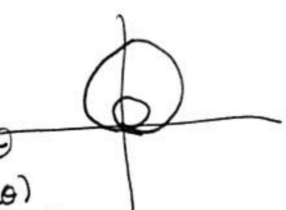
$7\pi/6,\ 11\pi/6$

$$\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (3+6\sin\theta)^2\, d\theta =$$

$$\frac{1}{2}\int_{7\pi/6}^{11\pi/6} 9 + 36\sin\theta + 36\sin^2\theta\, d\theta$$

$36\cdot\frac{1}{2}(1-\cos 2\theta)$

$$\frac{1}{2}\int_{7\pi/6}^{11\pi/6} 27 + 36\sin\theta - 18\cos 2\theta\, d\theta = \left[27\theta - 36\cos\theta - 9\sin 2\theta\right|_{7\pi/6}^{11\pi/6} =$$



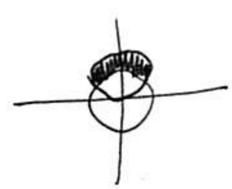
$$\frac{18\pi - 27\sqrt{3}}{2} = 9\pi - \frac{27}{2}\sqrt{3}$$

3. Find the area inside $r = 2\sin\theta$ and outside $r = 1$. Sketch the graph.

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \pi/6,\ 5\pi/6$



$$\frac{1}{2}\int_{\pi/6}^{5\pi/6} (2\sin\theta)^2 - 1^2\, d\theta = \int_{\pi/6}^{5\pi/6} 4\sin^2\theta - 1\, d\theta =$$

$4\cdot\frac{1}{2}(1-\cos 2\theta)$

$$\frac{1}{2}\int_{\pi/6}^{5\pi/6} 1 - 2\cos 2\theta\, d\theta = \theta - \sin 2\theta \Big|_{\pi/6}^{5\pi/6} = \frac{\sqrt{3} + 4\pi/6}{2} = \frac{2\pi/3 + \sqrt{3}}{2} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$