MTH 264, Quiz #12, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. For the differential equations below, determine if they are i) ordinary or partial, ii) linear or non-linear, iii) order.
   a. $\frac{dy}{dx} - ye^x = x$ ordinary, first order, linear

   b. $u_x - u_{yy} = u_x^2$ partial, second order, nonlinear

2. Use technology to sketch the direction field $\frac{dy}{dx} = 3x^2 - 2y^2 + xy$

See graph next page

3. Verify that the expression $y^2 - 2\ln y = x^2$ is a solution to the differential equation $\frac{dy}{dx} = \frac{xy}{y^2-1}$.

$$2yy' - \frac{2}{y}y' = 2x$$
$$y'\left(2y - \frac{2}{y}\right) = 2x$$
$$y' = \frac{2x}{2y - \frac{2}{y}} \cdot \frac{y}{y} = \frac{2xy}{2y^2 - 2} = \frac{xy}{y^2 - 1} \quad \checkmark$$

4. Use Euler's method to approximate the value of the solution to $y' = \frac{1}{3}x(2-y)$ at $y(4)$, starting at $y(0) = 1$, using $n = 8$ steps.

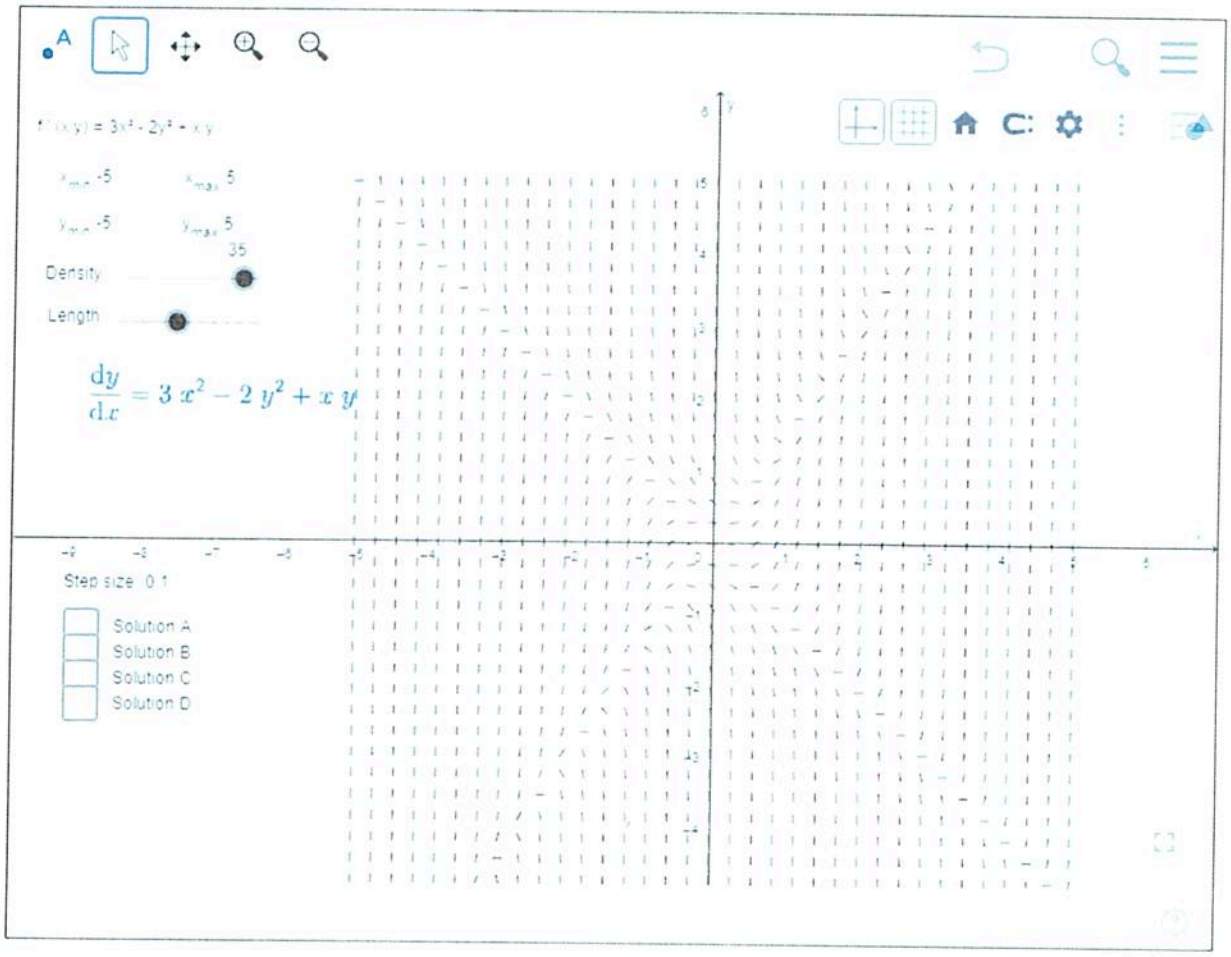
$\Delta x = \frac{4-0}{8} = 0.5$

$m_1 = \frac{1}{3}(0)(2-1) = 0$
$y_1 = 1$

$m_2 = \frac{1}{3}(0.5)(2-1) = \frac{1}{6}$
$y_2 = \frac{1}{6}(0.5) + 1 = \frac{13}{12}$

Continued in Excel

final point $(4, 1.95358\ldots)$


f(x,y) = 3x² - 2y² + x y
x_min -5
x_max 5
y_min -5
y_max 5
35
Density
Length
dy/dx = 3 x² − 2 y² + x y
Step size 0.1
Solution A
Solution B
Solution C
Solution D