4/25/2024

Conic Sections in Polar Coordinates (7.5)

Recall: Conic Sections in Rectangular Coordinates

Circle

$$(x-h)^2+(y-k)^2=r^2$$

Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Or

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

Or

$$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$$

Parabola

$$y-k=\pm 4a(x-h)^2$$

Or

$$x-h=\pm 4a(y-k)^2$$

In General form: $Ax^2+By^2+Cx+Dy+E=0$

Circle : A and B the same size and same sign
Ellipse: A and B different sizes, same sign
Hyperbola: A and B, different signs
Parabola: either A or B is zero (only one squared term)

Focus: c
Ellipse: $b^2+c^2=a^2$
Hyperbola: $a^2+b^2=c^2$

Eccentricity: $e=\frac{c}{a}$
Circle: $c=0\rightarrow e=0$
Ellipse: $a>c\rightarrow e\in(0,1)$
Parabola: $a=c\rightarrow e=1$
Hyperbola: $c>a\rightarrow e\in(1,\infty)$

Conics in Polar Form:

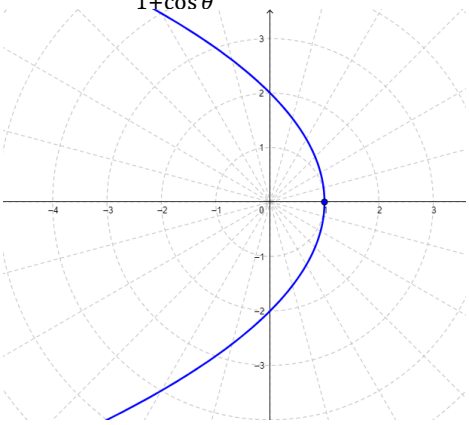
Circle:

$$r=a, r=2a\cos\theta\,, r=2a\sin\theta$$

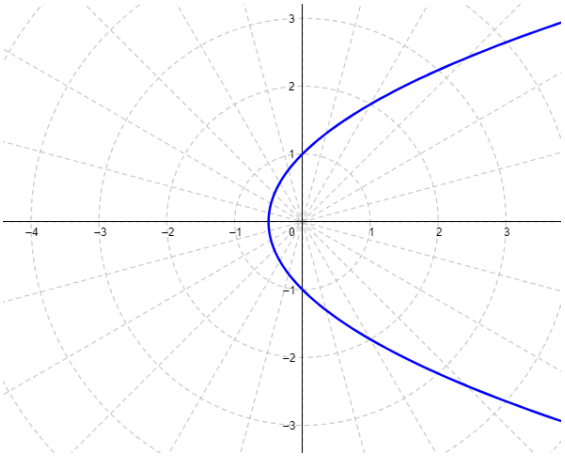
Ellipse/Hyperbola/Parabola:

$$r = \frac{a}{1 \pm e\cos\theta}, r = \frac{a}{1 \pm e\sin\theta}$$

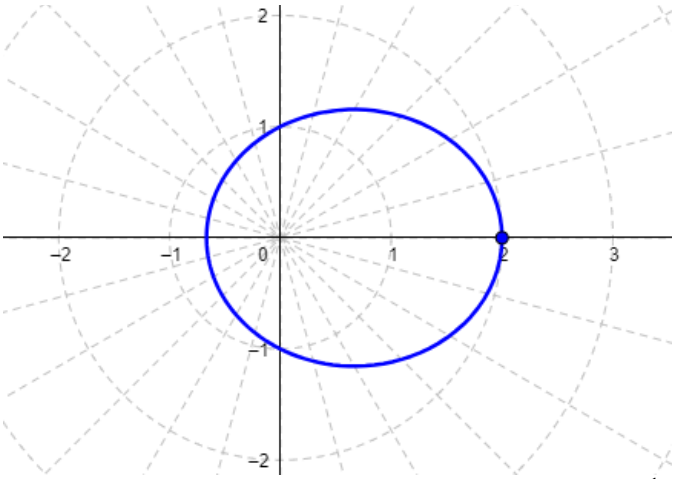
Graph $r = \frac{2}{1+\cos\theta}$



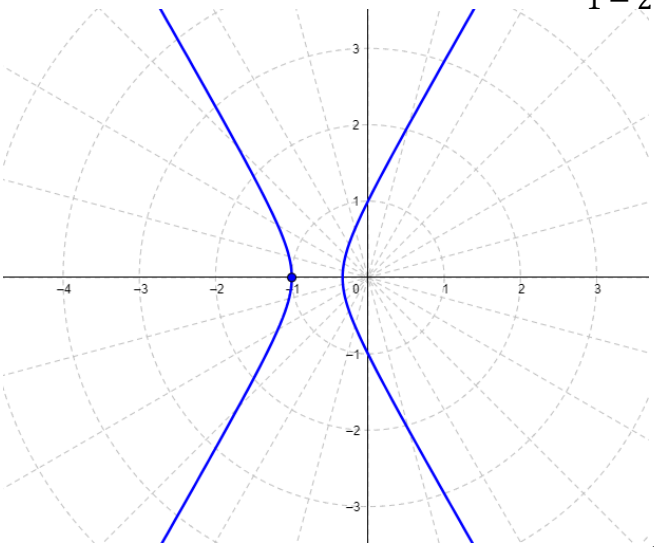
$$r = \frac{1}{1-\cos\theta}$$



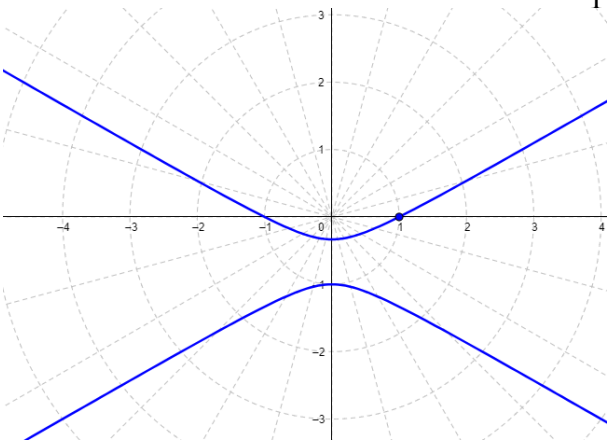
$$r = \frac{1}{1-\frac{1}{2}\cos\theta}$$



$$r = \frac{1}{1 - 2\cos\theta}$$



$$r = \frac{1}{1 - 2\sin\theta}$$



$$r = \frac{6}{2 + 3\cos\theta}$$

$$r = \frac{3}{1 + \frac{3}{2}\cos\theta}$$

$e = \frac{3}{2}$, hyperbola.