4/18/2024

Polar Coordinates

$$x = r\cos\theta\,, y = r\sin\theta$$
$$x^2 + y^2 = r^2, \tan^{-1}\left(\frac{y}{x}\right) = \theta$$

Convert the (1,1) from rectangular coordinates to polar coordinates.

$$r^2 = (1)^2 + (1)^2 = 2$$
$$r = \sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

$$\left(\sqrt{2}, \frac{\pi}{4}\right)$$

Equivalent points:

$$\left(\sqrt{2}, \frac{9\pi}{4}\right), \left(\sqrt{2}, -\frac{7\pi}{4}\right)$$
$$\left(-\sqrt{2}, \frac{5\pi}{4}\right), \left(-\sqrt{2}, -\frac{3\pi}{4}\right)$$

Convert $\left(4, \frac{\pi}{3}\right)$ in polar to rectangular coordinates.

$$x = 4\cos\left(\frac{\pi}{3}\right) = 4\left(\frac{1}{2}\right) = 2$$
$$y = 4\sin\left(\frac{\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$

$$\left(2, 2\sqrt{3}\right)$$

Angles should be in radians!!!

Convert the equation $y = 3x + 4$ into polar form.

$$r\sin\theta = 3r\cos\theta + 4$$

$$r\sin\theta - 3r\cos\theta = 4$$
$$r(\sin\theta - 3\cos\theta) = 4$$

$$r = \frac{4}{\sin\theta - 3\cos\theta}$$

Convert $(x+1)^2 + y^2 = 4$

$$x^2 + 2x + 1 + y^2 = 4$$

$$x^2 + y^2 + 2x = 3$$

$$r^2 + 2r\cos\theta = 3$$

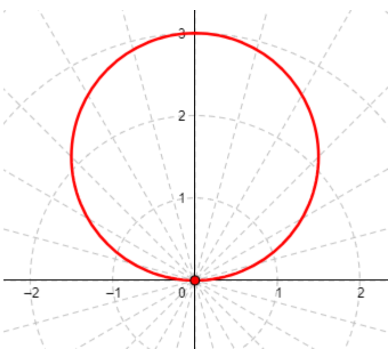
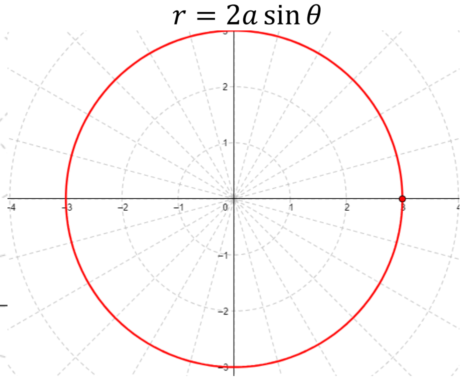
$$(x+2)^2 + y^2 = 4$$
$$x^2 + 4x + 4 + y^2 = 4$$
$$x^2 + y^2 = -4x$$
$$r^2 = -4r\cos\theta$$
$$r = -4\cos\theta$$

Convert $r = 6\sin\theta$ to rectangular

$$r^2 = 6r\sin\theta$$
$$x^2 + y^2 = 6y$$
$$x^2 + y^2 - 6y = 0$$
$$x^2 + (y^2 - 6y + 9) = 9$$
$$x^2 + (y-3)^2 = 9$$

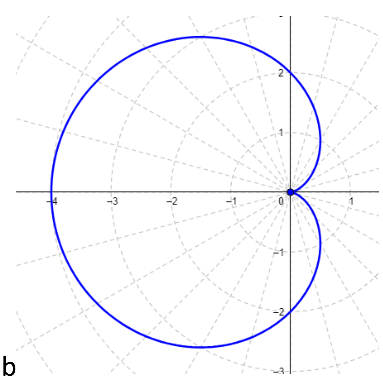
Circles

$$r = 3$$
$$r = 2\cos\theta$$
$$r = 2a\sin\theta$$

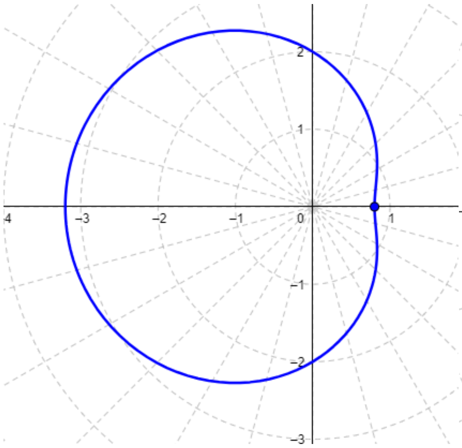
Cardioids and limacons

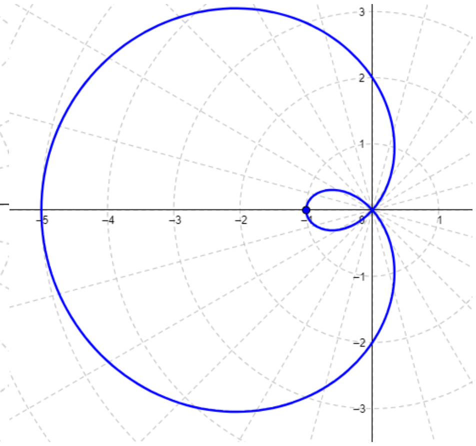
$$r = a \pm b\cos\theta$$
$$r = a \pm b\sin\theta$$



Cardioid when a=b

Limacon when $a \neq b$



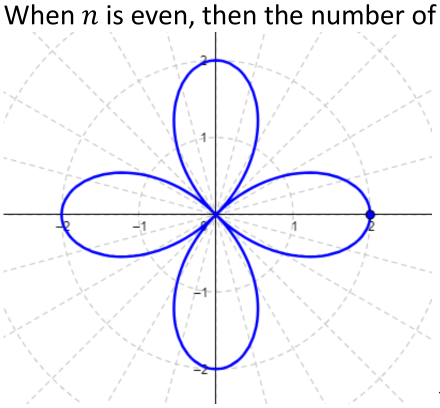


Rose

The number of petals will depend on the multiplier of $\theta$

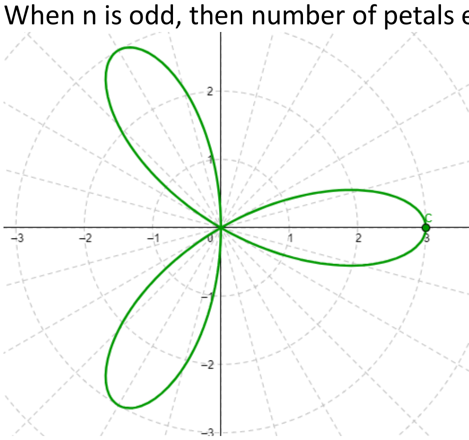
$$r = a\cos(n\theta)$$
$$r = a\sin(n\theta)$$

When $n$ is even, then the number of petals is equal to 2n



$r = 2\cos 2\theta$

When n is odd, then number of petals equals n



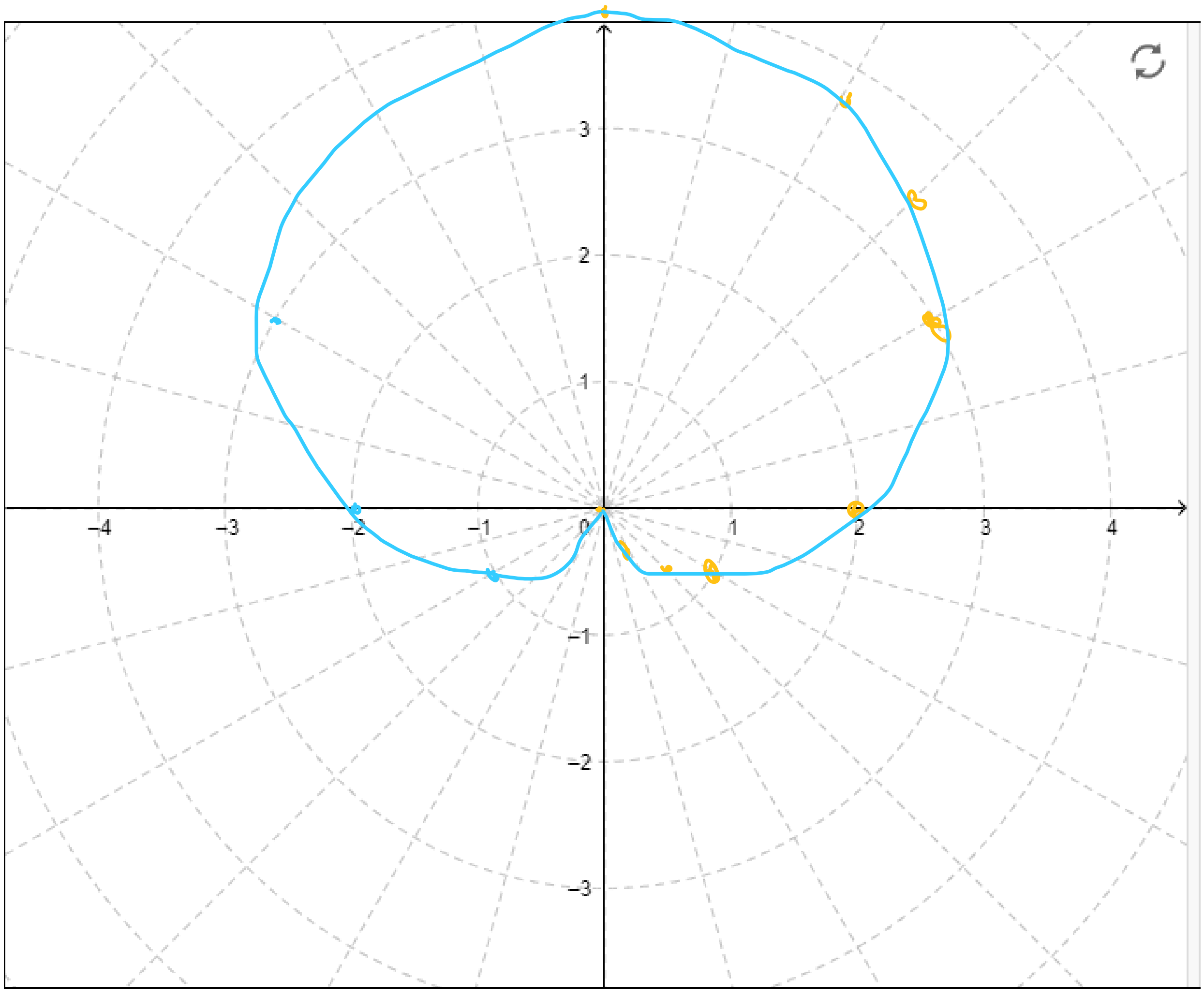
$r = 3\cos 3\theta$

Line through the origin is $\theta = constant$

https://www.geogebra.org/m/ApcfSCZY

Plot $r = 2 + 2\sin\theta$

| $\theta$ | $r$ |
|---|---|
| $0$ | $2 + 2(0) = 2$ |
| $\frac{\pi}{6}$ | $2 + 2\left(\frac{1}{2}\right) = 3$ |
| $\frac{\pi}{4}$ | $2 + 2\left(\frac{\sqrt{2}}{2}\right) = 2 + \sqrt{2} \approx 3.41$ |
| $\frac{\pi}{3}$ | $2 + 2\left(\frac{\sqrt{3}}{2}\right) = 2 + \sqrt{3} \approx 3.73$ |
| $\frac{\pi}{2}$ | $2 + 2(1) = 4$ |
| $-\frac{\pi}{6}$ | $2 + 2\left(-\frac{1}{2}\right) = 1$ |
| $-\frac{\pi}{4}$ | $2 + 2\left(-\frac{\sqrt{2}}{2}\right) = 2 - \sqrt{2} \approx 0.59$ |
| $\frac{-\pi}{3}$ | $2 + 2\left(-\frac{\sqrt{3}}{2}\right) = 2 - \sqrt{3} \approx 0.27$ |
| $-\frac{\pi}{2}$ | $2 + 2(-1) = 0$ |



3
2
1
0
−1
−2
−3
−4
4