

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Green Mountain Coffee Roasters produces many varieties of flavored coffees, teas and K-cups. The net sales S of the company have grown exponentially at the rate of 37.2% per year, and the growth can be approximated by $\frac{dS}{dt} = 0.372S$, where t is the number of years since 2005.

- a. Find the function that satisfies the equation, given that net sales in 2005 were approximately \$150,800.

$$S(t) = S_0 e^{0.372t} = 150800 e^{0.372t}$$

- b. Estimate the net sales in 2008, 2010, 2019.

$$S(3) \quad S(5) \quad S(14)$$

$$S(3) = 460,335 \quad S(5) = 968,700 \quad S(14) = 27,555,418$$

- c. What is the doubling time for $S(t)$?

$$\frac{\ln 2}{0.372} = t = 1.86 \text{ years.}$$

2. Iodine-131 has a decay rate of 9.6% per day. The rate of change of an amount N of iodine-131 is given by $\frac{dN}{dt} = -0.096N$, where t is the number of days since the decay began. Suppose that 500 g of iodine-131 was initially present.

- a. What is the equation that models the amount of iodine?

$$N(t) = 500 e^{-0.096t}$$

- b. How much will remain after 4 days?

$$N(4) = 500 e^{-0.096(4)} = 390.57 \text{ g}$$

- c. After how many days will half of original 500 g of iodine-131 remain?

$$\frac{\ln \frac{1}{2}}{-0.096} = t = 7.22 \text{ days}$$

3. The elasticity of demand is given by $E(x) = -\frac{x D'(x)}{D(x)}$. Find the elasticity for $D(x) = 300e^{-0.15x}$, at $x = 10$.

$$D' = -45e^{-0.15x}$$

$$E(x) = \frac{10(-45e^{-0.15 \cdot 10})}{300e^{-0.15 \cdot 10}} = \frac{10(-45)}{300} = -1.5$$

4. Integrate $\int \frac{2}{x} - 7e^{4x} + \sqrt{x^5} dx$. $\rightarrow x^{5/2}$

$$2 \ln x - \frac{7}{4} e^{4x} + \frac{2}{7} x^{7/2} + C$$