MTH 261, Quiz #12, Spring 2024 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Find the consumer's and producer's surplus for $D(x) = -3x + 7, S(x) = 2x + 2$.

$$-3x+7=2x+2 \qquad S(1)=2(1)+2=4$$
$$-5x=-5$$
$$x=1$$

$$\int_0^1 -3x+7-4\,dx = \int_0^1 -3x+3\,dx = -\frac{3}{2}x^2+3x\Big|_0^1 = -\frac{3}{2}+3 = \boxed{\frac{3}{2}}$$

$$\int_0^1 4-2x-2\,dx = \int_0^1 2-2x\,dx = 2x-x^2\Big|_0^1 = 2-1 = \boxed{1}$$

2. Find the accumulated present value of a continuous income stream if $B(t) = \int_0^T R(t)e^{-kt}dt$, where $R(t) = t^2, T = 20, k = 8\%$.

$$\int_0^{20} t^2 e^{-0.08t}\,dt$$

$$-\frac{t^2}{0.08}e^{-0.08t} - \frac{2t}{(0.08)^2}e^{-0.08t} - \frac{2}{(0.08)^3}e^{-0.08t}\Big|_0^{20} =$$

$$-3059.994 + \frac{2}{(0.08)^3}\cancel{e^{-.08(0)}} = 846.26$$

±	u	dv
+	t^2	$e^{-0.08t}$
−	$2t$	$\frac{-1}{0.08}e^{-0.08t}$
+	2	$\frac{1}{(0.08)^2}e^{-0.08t}$
−	0	$\frac{-1}{(0.08)^3}e^{-0.08t}$

3. Integrate $\int_{-1}^{1}\int_{x}^{1} xy^2dydx$.

$$\int_{-1}^1 x\left[\frac{1}{3}y^3\Big|_x^1\right]dx = \int_{-1}^1 x\left[\frac{1}{3}-\frac{x^3}{3}\right]dx = \int_{-1}^1 \frac{x}{3}-\frac{x^4}{3}\,dx =$$

$$\frac{1}{6}x^2 - \frac{1}{15}x^5\Big|_{-1}^1 = \cancel{\frac{1}{6}} - \frac{1}{15} - \cancel{\frac{1}{6}} + \frac{1}{15}(-1)^5 = -\frac{2}{15}$$

4. Find the value of k so that $f(x) = kx^2$ on the interval $[1,4]$ so the $f(x)$ is a probability density function.

$$K\int_1^4 x^2dx = \frac{K}{3}x^3\Big|_1^4 = \frac{K}{3}[64-1] = \frac{63}{3}K = 21K = 1$$

$$K = \frac{1}{21}$$