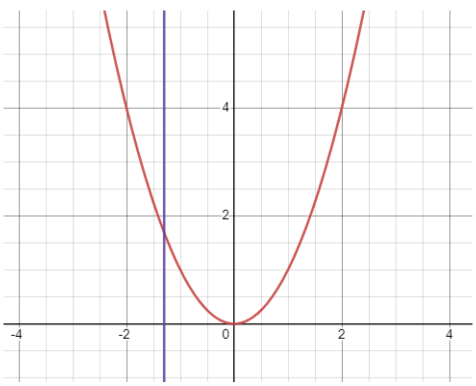
1/17/2024

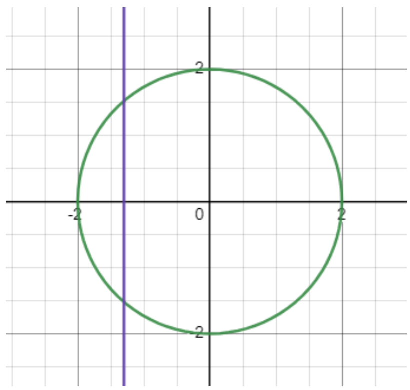
Introduction to the course
Algebra Review (functions, operations on functions; linear, quadratic/polynomial, rational, exponential, log functions)
Functions of Several Variables

What is a function:
A function is an equation where one variable depends on another variable in such a way that the function variable has only one possible value for each input value.

Vertical line test: draw a graph of the equation, and if no vertical line crosses the graph in more than once place, then the equation is a function.

https://www.desmos.com/calculator





If you can solve for y, then it’s a function.

$$f(x) = x^3 - 2$$

Function notation tells you which variable is the independent variable (the input variable), and the formula you use to obtain the value. $y = x^3 - 2$

Can be use to tell us to evaluate a function at a particular input value:

$f(2)$ is saying take the function f, and replace x with 2 everywhere.

$$f(2) = 2^3 - 2 = 8 - 2 = 6$$

$$f(t) = t^3 - 2$$

$$f(a + h) = (a + h)^3 - 2$$

Common function types:

Identity function $f(x) = x$
Square Function $f(x) = x^2$

Cubic Function: $f(x) = x^3$
Absolute value function $f(x) = |x|$ (abs(x))
Square root function: $f(x) = \sqrt{x}$
Cube root function: $f(x) = \sqrt[3]{x}$ (beware of $f(x) = x\sqrt[3]{x}$… this is x times cube-root-of-x, not x-cubed times square root of x… $f(x) = x^3\sqrt{x}$)
The reciprocal function $f(x) = \frac{1}{x}$
Exponential functions $f(x) = e^x$
Logarithmic functions $f(x) = \log x$, $f(x) = \ln x$, $f(x) = \log_a x$

Domain and range of functions
Domain is the set of input values (x) that when put into a function produces a defined result.
Range is the set of outputs of the function

For example: $f(x) = x^2$, the domain is all real numbers, the range is $[0, \infty)$ because the smallest value the square be from a real input is 0, and it can be as large as we like.

Another example: $f(x) = \frac{1}{x}$. The domain is all reals except 0: $\{x|x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$. The range is also all reals except 0: $\{y|y \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$.

Operation on functions:

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$

$$(f \circ g)(x) = f\big(g(x)\big)$$

If $f(x) = x^2 + 1$, $g(x) = x - 2$

$$(f+g)(x) = x^2 + 1 + x - 2 = x^2 + x - 1$$
$$(fg)(x) = (x^2+1)(x-2) = x^3 - 2x^2 + x - 2$$
$$f\big(g(x)\big) = (x-2)^2 + 1$$

Inverse functions: $f^{-1}(x) \neq \frac{1}{f(x)}$

Is a function like f(x) but where f(x) is the x value and x is the y-value.
To find an inverse, swap x and y in the equation, and then solve for y.

$$f(x) = x^3 - 2$$

$$y = x^3 - 2$$
$$x = y^3 - 2$$

$$x + 2 = y^3$$
$$y = \sqrt[3]{x+2}$$

$$f^{-1}(x) = \sqrt[3]{x+2}$$

This is the inverse of f(x).

In order for the inverse to be a function, the original function must pass the horizontal line test (it must be one-to-one).

The square function is not one-to-one, but the cube function is.
If the function is not one-to-one, then you will have to restrict the domain to obtain an inverse function.

Transformations of functions:
Shifts: horizontal shift: replace x with x-c moves the graph c units right, x+c moves the graph c units left
Vertical shift: $f(x) + c$ moves the graph c units up, and $f(x) - c$ move the graph c units down

Stretching and compressing:
Vertical stretch: $kf(x)$ if k>1, vertical compression is $kf(x)$ if 0<k<1
Horizontal stretch: f(kx) is a horizontal stretch if 0<k<1, and f(kx) is horizontal compression is k>1

Reflections:
Reflecting vertically is $-f(x)$
Reflecting horizontally is $f(-x)$

Linear functions:
Equations of lines:

$$y = mx + b$$
$$Ax + By = C$$
$$(y - y_1) = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Laws of exponents / laws of logs

$$x^a x^b = x^{a+b}$$
$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{1}{x^n} = x^{-n}$$
$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$(x^a)^b = x^{ab}$$

$$\log(MN) = \log(M) + \log(N)$$

$$\log\left(\frac{M}{N}\right) = \log(M) - \log(N)$$

$$\log(M^r) = r\log(M)$$

Quadratic equation: the solution to $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rational functions: $R(x) = \frac{P(x)}{Q(x)}, Q(x) \neq 0$

To be a rational function, P(x) and Q(x) are both polynomials

Rational functions can have vertical asymptotes (or holes) where the denominator is equal to 0. Horizontal asymptote is 0 is the degree of the numerator is less than the degree of the denominator. If they have the same degree, the horizontal asymptote is the constant obtained from dividing the leading terms.

If the numerator is a greater degree, do long division. One degree higher, then you get an oblique asymptote (or slant asymptote).

Piecewise functions:
https://www.graphfree.com/

Functions of more than one variable

A function where there is more than one independent input variable.

$$z = f(x, y)$$

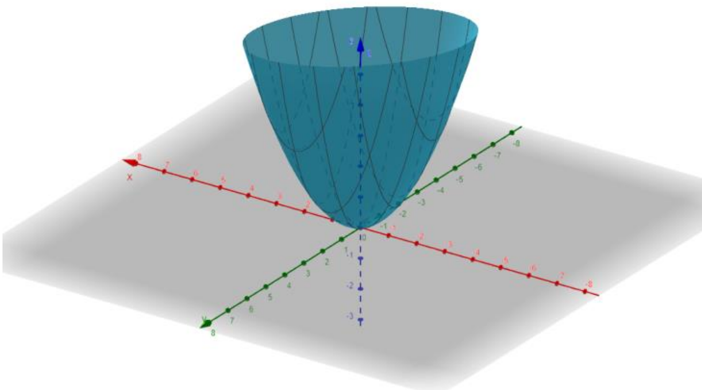
$$w = f(x, y, z)$$

$$q = f(u, v, x, y, z, w)$$

$$f(x, y) = x^2 - xy + y^2$$

$$f(2,1) = (2)^2 - (2)(1) + (1)^2 = 4 - 2 + 1 = 3$$

Represent a point in space (2,1,3)
https://www.geogebra.org/3d?lang=en



Domain and range:
The range is going to behave very similarly to the one-variable functions.

Example: find the domain and range of

$$f(x,y) = x^2 - xy + y^2$$

Domain is all real numbers in both x and y
Range: $[0, \infty)$

Example: find the domain and range of $f(x,y) = \ln(2x - 3y + 1)$

Domain: $\{(x,y) | 2x - 3y + 1 > 0\}$

Range : $(-\infty, \infty)$ or all reals