MTH 261, Exam #2, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. Find the absolute extrema of the function $f(x) = x^3 - 2x^2 + 5$ on the interval $[-2,4]$. (10 points)

$f'(x) = 3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0,\ x = 4/3$

$f(-2) = -11$ ← absolute min

$f(0) = 5$

$f(4/3) = 3.8148\ldots$

$f(4) = 37$ ← absolute max

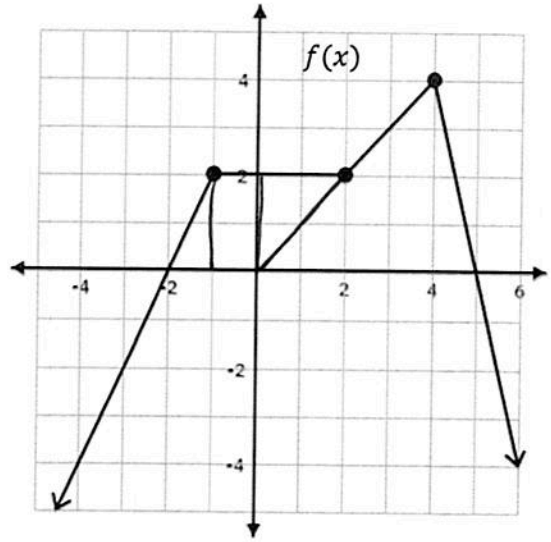
2. Find $f(x)$ if $f'(x) = 8x^3 + 4x^2 - 2$. Find the constant of integration if $f(0) = 4$. (10 points)

$\int 8x^3 + 4x^2 - 2\,dx = 2x^4 + \frac{4}{3}x^3 - 2x + C$

$C = 4$

$f(x) = 2x^4 + \frac{4}{3}x^3 - 2x + 4$

3. Use graph to the right to evaluate the integral $\int_{-2}^{5} f(x)dx$ geometrically. (12 points)



$\frac{1}{2}(2)(1) + \frac{1}{2}(5)(4) + 2 + \frac{1}{2}(2)(2)$

$= 1 + 10 + 2 + 2 = 15$

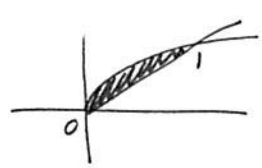
4. Find the area under the curve $f(x) = -x^3 + 4x$ over the interval $[0,2]$. (10 points)

$\int_0^2 -x^3 + 4x\,dx = -\frac{1}{4}x^4 + 2x^2 \Big|_0^2 = -\frac{1}{4}(16) + 2(4) =$

$-4 + 8 = 4$

5. Find the area bounded by $f(x) = x$ and $g(x) = \sqrt[4]{x}$. Sketch the graph. (10 points)

$\int_0^1 x^{1/4} - x\,dx = \frac{4}{5}x^{5/4} - \frac{1}{2}x^2 \Big|_0^1$



$= \frac{4}{5} - \frac{1}{2} = 0.8 - 0.5 =$

$0.3 = \frac{3}{10}$

6. Riverside Appliances is marketing a new refrigerator. It determines that in order to sell x refrigerators, the price per suit must be $p = 280 - 0.5x$. It also determines that the total cost of producing x refrigerators is given by $C(x) = 6000 + 0.6x^2$.

a. Find the total revenue $R(x)$. (5 points)

$R(x) = 280x - 0.5x^2$

b. Find the total profit $P(x)$. (5 points)

$P = R - C =$

$280x - 0.5x^2 - 6000 - 0.6x^2 = 280x - 1.1x^2 - 6000$

c. How many refrigerators must the company produce and sell in order to maximize profit? (6 points)

$P'(x) = 280 - 2.2x = 0$

$\frac{280}{2.2} = x \approx 127$ (must be a whole #)

d. What is the maximum profit? (5 points)

$P(127) = 11,818.1$

e. What is the price per refrigerator that must be charged in order to maximize profit? (6 points)

$p(127) = \$216.5$

7. Suppose that the price p in dollars and number of sales x of a certain item follow the equation $5p + 4x + 2px = 60$. Suppose also that p and x are both functions of time, measured in days. Find the rate at which x is changing $\left(\frac{dx}{dt}\right)$ when $x = 4, p = 5, \frac{dp}{dt} = 2$. (12 points)

$$5\frac{dp}{dt} + 4\frac{dx}{dt} + 2\frac{dp}{dt}x + 2p\frac{dx}{dt} = 0$$

$$\frac{dx}{dt}(4 + 2p) = \frac{dp}{dt}(-5-2x)$$

$$\frac{dx}{dt} = \frac{dp}{dt}\left(\frac{-5-2x}{4+2p}\right)$$

$$= 2\left(\frac{-5-2(4)}{4+2(5)}\right) = 2\left(\frac{-5-8}{4+10}\right) = 2\left(\frac{-13}{14}\right) = -\frac{13}{7}$$

8. Suppose that P_0 is invested in the Mandelbrot Bond Fund for which interest is compounded continuously at 4.8% per year. That is the balance P grows at the rate given by $\frac{dP}{dt} = 0.048P$.

a. Find the function that satisfies the equation in terms of P_0 and 0.048. (5 points)

$$P(t) = P_0 e^{0.048t}$$

b. Suppose that \$8000 is invested. What is the balance in the account after 1 year? (5 points)

$$P(t) = 8000e^{0.048t}$$

$$P(1) = 8000e^{0.048} = \$8393.37$$

c. What is the balance in the account after 3 years? (6 points)

$$P(3) = 8000e^{0.048\times 3} = \$9239.07$$

d. When will an investment of \$8000 double itself? (6 points)

$$\frac{\ln 2}{0.048} = 14.44 \text{ years}$$

9. Carbon-14 has a decay rate that is modeled by the equation $\frac{dN}{dt} = -0.00012097N$, where t is in years. How old is an ivory tusk if 35% of its original Carbon-14 remains? (12 points)

$$\frac{\ln(0.35)}{-0.00012097} = 8678.37 \text{ years}$$

10. The elasticity of demand is given by $E(x) = -\frac{xD'(x)}{D(x)}$. Find the elasticity for $D(x) = \sqrt{600 - x}$, at $x = 200$. (10 points)

$$(600-x)^{1/2}$$

$$D'(x) = \frac{1}{2}(600-x)^{-1/2}(-1) = \frac{-1}{2\sqrt{600-x}}$$

$$E(x) = \frac{-200\left(\frac{-1}{2\sqrt{600-x}}\right)}{\sqrt{600-x}} = \frac{+100}{600-x} \Rightarrow \frac{100}{600-200} = \frac{100}{400} = \frac{1}{4}$$

11. Approximate the area under the curve $f(x) = \frac{1}{\sqrt{x}}$ on the interval $[1,9]$ by computing the area under 8 rectangles (using the right-hand rule). (15 points)

$\Delta x = \frac{9-1}{8} = 1 \qquad x_0 = 1,\ x_1 = 2, \ldots\ x_8 = 9$

$A_i = f(x_i)\Delta x \qquad A = \sum_{i=1}^{8} f(x_i)\Delta x$

$A = 1\left[1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{8}}\right] \approx 4.3714368\ldots$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

12. Find the equation of the tangent line to the implicitly defined function $x^4 - x^2y^3 = 12$ at the point $(-2,1)$. (12 points)

$4x^3 - 2xy^3 - x^2 3y^2 y' = 0$

$\frac{4x^3 - 2xy^3}{3x^2y^2} = \frac{dy}{dx} \rightarrow \frac{4(-2)^3 - 2(-2)(1)^3}{3(-2)^2(1)^2} = \frac{-32+4}{12}$

$y - 1 = -\frac{7}{3}(x+2) \qquad = \frac{-28}{12} = -\frac{7}{3}$

13. Integrate $\int \frac{4}{\sqrt{x}} + \frac{3}{2}e^{5x} - \frac{13}{x}\,dx$. (8 points)

$4x^{-1/2}$

$= 2 \cdot 4x^{1/2} + \frac{3}{10}e^{5x} - 13\ln x + C$

$8\sqrt{x} + \frac{3}{10}e^{5x} - 13\ln x + C$

14. Find the error(s) in the following work. Identify each error, explain why it is incorrect, and make an appropriate correction. [Hint: there is at least one mistake.] (10 points)

$$\int_1^2 x^2 - e^x dx = \left[\frac{1}{3}x^3 - \frac{e^{x+1}}{x+1}\right]_1^2$$

exponential functions don't use the power rule

$$= \left(\frac{8}{3} - \frac{e^3}{3}\right) - \left(\frac{1}{3} - \frac{e^2}{2}\right)$$

$$= \frac{7}{3} - \frac{e^2}{6}$$

$e^3 \neq e^2$ are not the same value

sign

$\frac{1}{2}$ bigger than $\frac{1}{3}$ so positive

$$\int_1^2 x^2 - e^x dx = \frac{1}{3}x^3 - e^x \Big|_1^2 = \frac{1}{3}(8) - e^2 - \frac{1}{3} + e$$

$$= \frac{7}{3} - e^2 + e$$

15. Integrate. (10 points each)

a. $\int x^2 e^x dx$

$x^2e^x - 2xe^x + 2e^x + C$

+/-	u	dv
+	x^2	e^x
−	$2x$	e^x
+	2	e^x
−	0	e^x

b. $\int x \ln x \, dx$

$u = \ln x$ $\quad dv = x\,dx$

$du = \frac{dx}{x}$ $\quad v = \frac{1}{2}x^2$

$$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

c. $\int \frac{e^{1/x}}{x^2}\,dx$

$u = 1/x$

$du = -\frac{1}{x^2}\,dx$

$$\int -e^u\,du = -e^u + C = -e^{1/x} + C$$

d. $\int \frac{1}{1-2x}\,dx$

$u = 1-2x$

$du = -2\,dx$

$$-\frac{1}{2}\ln|1-2x| + C$$