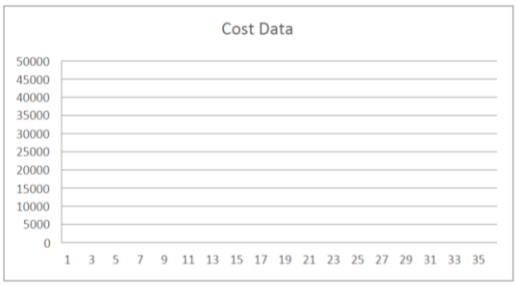
2/11/2023

Probability Topics
Counting
Two-Way tables

Quiz #2 Review
Don't say "see Excel". Why not? Because of this:

47317
41172
43974
44290
29297
47244
43185
42658
39178
41198
43505
35805
39181
40248
28157
34761
45148
33447



Probability Rules
Probabilities are proportions: all values are between 0 and 1.
All probabilities that describe the outcome of a single event must add to 1.

A 0 probability means that an event is impossible. (like rolling a 7 on a standard die)
A 1 probability means that an event always occurs.

You can express probabilities as decimals, fractions, or percentages, or very small ones may be in scientific notation.

In Excel, a number in scientific notation will display like 3.5E-05.
This is equivalent to 3.5×10^{-5}.
This is equivalent to 0.000035.
Don't just write 3.5. This number is not a probability.

A probability distribution is a set of outcomes and their likelihoods that represent all possible outcomes for an event, and the corresponding probabilities for those events. For the discrete case, these are typically given in tables.

x	1	2	3	4	5	6
$p(x)$	0.10	0.20	0.15	0.12	0.21	0.22

Is this a probability distribution?
Are there any values in the table that are less than 0?
Are there any values in the table that are bigger than 1?
Do all the probabilities add up to 1?
Since all three are true, this is a probability distribution.

A complement?

The complement is the non-event… If the event is A, then the complement is not A.

What is the probability of getting a 2 on my weighted die?

$$P(X = 2) = P(2) = 0.2, or\ 20\%$$

What is the probability of NOT getting a 2?

$$P(X \neq 2) = ?$$

Since all probabilities have to add to 1, then I can use this fact to get this probability.

$$P(X = 2) + P(X \neq 2) = 1$$
$$P(X \neq 2) = 1 - P(2)$$

$$P(X \neq 2) = 1 - 0.2 = 0.8, or\ 80\%$$

x	1	2	3	4	5	6
$p(x)$	0.10	0.20	0.15	0.12	0.21	?

Given this information on the values from 1-5, what does the probability for 6 have to be to make this a probability distribution?

If I add up all the given probabilities and subtract the sum from 1, this will tell me what the remaining probability has to be, to make this a valid probability distribution.

All the values in the table are mutually exclusive: which means that only one outcome can happen at a time.

If I want to know the probability of two (or more) different outcomes, then I add the probabilities for the individual outcomes together.

$$P(X = 2\ or\ X = 3) = P(2) + P(3)$$

We can also talk about ranges of values: what is the probability of getting an outcome less than 3? Or greater than or equal to 2?

$$P(X < 3) = P(0) + P(1) + P(2)$$
$$P(X \geq 2) = P(2) + P(3) + P(4) + P(5) + P(6)$$

We could also do in-between outcomes like between 1 and 4…

$$P(1 < X < 4) = P(2) + P(3)$$
$$P(1 \leq X \leq 4) = P(1) + P(2) + P(3) + P(4)$$

In discrete cases, the inequalities used matter a lot because it determines whether the end value is included or not.

Another kind of example of a probability problem is a bowl of marbles.

Suppose I have a bowl of marbles and I know that there are 4 red marbles, 5 green marbles, 6 blue marbles and 7 orange marbles.

How many marbles are there in the bowl? 4+5=9, 6+7=13, 9+13=22

What is the probability of selecting a green marble? 5/22
What is the probability of NOT selecting a green marble? 17/22
What is the probability of selecting either a red or blue marble? 10/22
What is the probability of selecting a clear marble? 0
What is the probability of selecting a green marble followed by an orange marble? 5/22 * 7/21 =5/22*1/3= 5/66
(without replacement = you are still holding onto the green marble, and so there are fewer marbles in the bowl)
(with replacement = you put the first one back before picking the next thing, keeping the number of marbles in the bowl the same)

Counting Rules
Multiplication Rule, Permutations, Combinations

Multiplication Rule: separate events, I calculate the number of outcomes for each event, and then multiply the number of outcomes together.

Suppose I flip a coin, and I do it 5 times. How many possible strings of outcomes are possible?
2*2*2*2*2=32
HHHHH, HHHHT, HHHTH, HHTHH,…etc. there are 32 possible combinations of outcomes (in order).

These event are called simple events: only one string of outcomes is in each "event". The probability of getting HTHHT, is 1/32

A compound event combines a set of simple outcomes: The probability of getting one tail: HHHHT, HHHTH, HHTHH, HTHHH, THHHH… 5/32

Wardrobes: you are going vacation and you are taking 3 pairs of pants, 4 shirts, 2 jackets, and 2 pairs of shoes. How many possible outfits can you construct without exactly repeating any combination?
3*4*2*2=48

License plates. Suppose a state license is of the form LLL-###. How many possible license plates are there?
26*26*26*10*10*10=17,576,000.

What is the probability that our license plate contains only vowels and even numbers?
5*5*5*5*5*5 = 15,625

The probability is 15625/17576000 = 8.89x10^-4 = 0.000889…

Permutations
A t-ball team has 14 players and you need to pick 10 for the field:
Like a multiplication rule (we can think of it that way), we can't reuse an option once we pick it.
14*13*12*11*10*9*8*7*6*5=3,632,428,800.

$$P(n,r) = nPr = \frac{n!}{(n-r)!}$$

$$14P10 = P(14,10) = \frac{14!}{(14-10)!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

! is called a "factorial"
7! = 7*6*5*4*3*2*1 = 5040

Combinations
Not only can't we reuse the outcomes, but we also don't care about the order.
Suppose your boss is giving away identical year-end bonuses to three randomly selected employees. There are 15 employees at the company. How many ways are there to choose people to give the bonuses?

$$nCr = C(n,r) = \frac{n!}{(n-r)!\,r!}$$

$$15C3 = \frac{15!}{(15-3)!\,3!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1}$$

$$= \frac{15 \times 14 \times 13}{6} = 5 \times 7 \times 13 = 455$$

Combination notation: $nCr = \binom{n}{r}$

The thing is that 0! = 1