MTH 263, Quiz #16, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Set up the integral to find the arc length of the curve $y = x^{2/3}$ on the interval [1,8]. Use technology to evaluate it and report your answer to 4 decimal places.

$y' = \frac{2}{3}x^{-1/3}$

$$s = \int_1^8 \sqrt{1 + \left(\tfrac{2}{3}x^{-1/3}\right)^2}\, dx = \int_1^8 \sqrt{1+\frac{4}{9x^{2/3}}}\, dx =$$

$$\int_1^8 \frac{\sqrt{9x^{2/3}+4}}{\sqrt{9x^{2/3}}}\, dx = \int_1^8 \frac{1}{3x^{1/3}}\sqrt{9x^{2/3}+4}\, dx$$

$u = 9x^{2/3}+4$

$du = \cancel{9}^{3}\,\frac{2}{3}x^{-1/3}\,dx$

$du = 6x^{-1/3}dx$

$\frac{1}{6}du = x^{-1/3}dx$

$$= \frac{1}{3}\int \frac{1}{6} u^{1/2}\, dx = \frac{1}{\cancel{18}}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|_{x=1}^{x=8} = \frac{1}{27}\left(\sqrt{9x^{2/3}+4}\right)^3\Big|_1^8 =$$

$$\frac{1}{27}\left(\left(\sqrt{9(4)+4}\right)^3 - \left(\sqrt{9+4}\right)^3\right) = \boxed{\frac{40\sqrt{40} - \sqrt{13}\cdot 13}{27}} \approx 7.6337$$

2. Find the surface area if the curve $y = \frac{1}{x}$ on the interval $[\frac{1}{2}, 1]$ is rotated around the y-axis.

$y' = -x^{-2}$

$$2\pi\int_{1/2}^1 x\sqrt{1+\left(\tfrac{-1}{x^2}\right)}\, dx = 2\pi\int_{1/2}^1 x\sqrt{1+\frac{1}{x^4}}\, dx =$$

$$2\pi\int_{1/2}^1 x\sqrt{\frac{x^4+1}{x^4}}\, dx = 2\pi\int_{1/2}^1 \frac{x}{x^2}\sqrt{x^4+1}\, dx = 2\pi\int_{1/2}^1 \frac{1}{x}\sqrt{x^4+1}\, dx$$

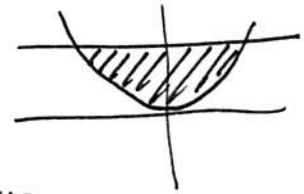
used calculator to integrate

$2\pi * 0.798388 \approx 5.016$

3. A force of $F = 20x - x^3$ stretches a nonlinear spring by x meters. What work is required to stretch the spring from x=0 to x=2 meters?

$$W = \int_0^2 20x - x^3\, dx = 10x^2 - \frac{1}{4}x^4\Big|_0^2 = 40 - 4 = 36$$

4. Find the center of mass $(\bar{x}, \bar{y})$ for the region bounded by $y = \frac{5}{4}x^2, y = 5$ assuming constant density.



$$M = \rho \int_{-2}^{2} 5 - \tfrac{5}{4}x^2\, dx = \rho \left[5x - \tfrac{5}{12}x^3 \right|_{-2}^{2}$$

$$\rho \left[5(2) - \tfrac{5}{12}(8) - \left(5(-2) - \tfrac{5}{12}(8) \right) \right] = \tfrac{40}{3}\rho$$

$$M_x = \frac{\rho}{2} \int_{-2}^{2} 5^2 - \left(\tfrac{5}{4}x^2 \right)^2 dx = \frac{\rho}{2} \int_{-2}^{2} 25 - \tfrac{25}{16}x^4\, dx =$$

$$\frac{\rho}{2} \left[25x - \tfrac{5}{16}x^5 \right|_{-2}^{2} = \frac{\rho}{2} \left[25(2) - \tfrac{5}{16}(2)^5 - \left(25(-2) - \tfrac{5}{16}(-2)^5 \right) \right] =$$

$$40\rho$$

$$M_y = \rho \int_{-2}^{2} x\left(5 - \tfrac{5}{4}x^2\right) dx = \rho \int_{-2}^{2} 5x - \tfrac{5}{4}x^3\, dx =$$

$$\rho \left[\tfrac{5}{2}x^2 - \tfrac{5}{16}x^4 \right|_{-2}^{2} = \rho \left[\tfrac{5}{2}(4) - \tfrac{5}{16}(16) - \left(\tfrac{5}{2}(-2)^2 - \tfrac{5}{16}(-2)^4 \right) \right]$$

$$= 0$$

$$\bar{X} = \frac{M_y}{M} = \frac{0}{40/3\,\rho} = 0$$

$$\bar{Y} = \frac{M_x}{M} = \frac{48\cancel{\rho}}{40/3\,\cancel{\rho}} = \frac{\cancel{40}\cdot 3}{\cancel{40}} = 3$$

$(0, 3)$ center of mass