4/12/2022

Review for Exam #2

Requests:
~~Differentials~~
~~Newton's Method/Excel~~
Deciding integration method
~~Applied Optimization~~
~~Quiz #10, problem 1~~
End of chapter 5

Differentials
(way back in Chapter 4 (4.2))
Suppose I want to estimate the value of $\sqrt{79.5}$ using differentials (linear approximation).

$$f(x+\Delta x) \approx y + \Delta y = y + f'(x)\Delta x$$

$$f(x) = \sqrt{x}$$

Nice value of x is 81. Then $\Delta x = 79.5 - 81 = -1.5$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Delta y = f'(x)\Delta x = \frac{1}{2\sqrt{81}}(-1.5) = \frac{1}{2(9)} \times \left(-\frac{3}{2}\right) = -\frac{1}{12}$$

$$f(79.5) \approx f(81) + \Delta y = 9 - \frac{1}{12} = 8\frac{11}{12} = 8.9166666\ldots$$

Applied Optimization, Quiz #10, problem #1

Find the closest point on the curve of $f(x) = x^2 + x$ to the point (1,5).

$$c = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x-1)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (x^2+x-5)^2}$$

$$c = \sqrt{x^2 - 2x + 1 + x^4 + x^3 - 5x^2 + x^3 + x^2 - 5x - 5x^2 - 5x + 25}$$

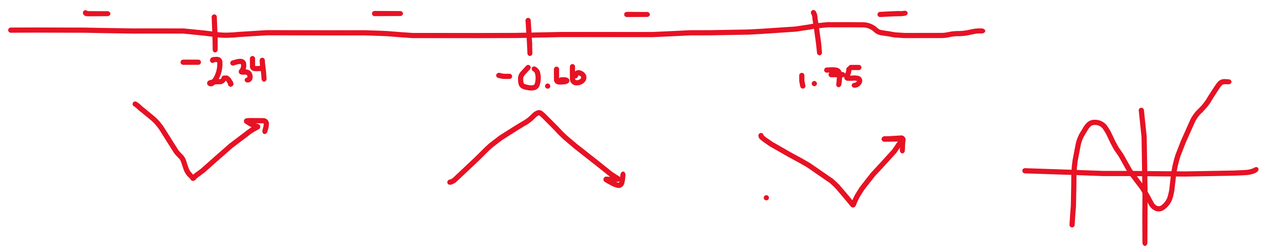
$$c = \sqrt{x^4 + 2x^3 - 8x^2 - 12x + 26}$$

$$c' = \frac{1}{2}(x^4 + 2x^3 - 8x^2 - 12x + 26)^{-\frac{1}{2}}(4x^3 + 6x^2 - 16x - 12) = \frac{2x^3 + 3x^2 - 8x - 6}{\sqrt{x^4 + 2x^3 - 8x^2 - 12x + 26}}$$

c' will be zero when the numerator is zero.

$$2x^3 + 3x^2 - 8x - 6 = 0$$

$$x \approx -2.34, -0.659, 1.75$$



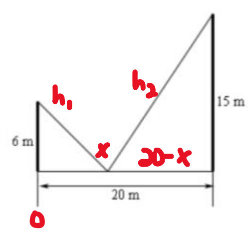
$$c(-2.34) = 3.8 \ldots$$
$$c(-0.659) = 5.4 \ldots$$
$$c(1.75) = 0.77 \ldots$$

Distance from function to point (to find the closest value)
Then plug into $f(x)$ to find the coordinate.

$$f(1.75) \approx 2.625$$

The point on the curve closest to (1,5) is approximately (1.75, 2.625)

Example 6 Two poles, one 6 meters tall and one 15 meters tall, are 20 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



Show Solution ▸

Function we want to minimize is the sum of the two hypotenuses. $f(x) = h_1 + h_2$

$$h_1 = \sqrt{x^2 + 36}, h_2 = \sqrt{(20 - x)^2 + 225} = \sqrt{400 - 40x + x^2 + 225} = \sqrt{x^2 - 40x + 625}$$

$$f(x) = \sqrt{x^2 + 36} + \sqrt{x^2 - 40x + 625}$$

To minimize, find the derivative and set it equal to zero.

$$f'(x) = \frac{2x}{2\sqrt{x^2 + 36}} + \frac{2x - 40}{2\sqrt{x^2 - 40x + 625}} = 0$$

$$\frac{-2x}{2\sqrt{x^2 + 36}} = \frac{2x - 40}{2\sqrt{x^2 - 40x + 625}}$$

$$\frac{-x}{\sqrt{x^2 + 36}} = \frac{x - 20}{\sqrt{x^2 - 40x + 625}}$$

$$x\sqrt{x^2 - 40x + 625} = (x - 20)\sqrt{x^2 + 36}$$

$$x^2(x^2 - 40x + 625) = (x - 20)^2(x^2 + 36)$$

$$x^4 - 40x^3 + 625x^2 = (x^2 - 40x + 400)(x^2 + 36)$$

$$x^4 - 40x^3 + 625x^2 = x^4 + 36x^2 - 40x^3 - 1440x + 400x^2 + 14400$$
$$189x^2 + 1440x - 14400 = 0$$

$$21x^2 + 160x - 1600 = 0$$
$$x = \frac{-160 \pm \sqrt{160^2 - 4(21)(-1600)}}{2(21)} = \frac{-160 \pm \sqrt{160000}}{42} = \frac{-160 \pm 400}{42}$$

$$x = \frac{40}{7}, -\frac{40}{3}$$

$x = \frac{40}{7}$ discard the other one because it's not between 0 and 20.

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = 2x^3 + 3x^2 - 8x - 6 = 0$$
$$f'(x) = 6x^2 + 6x - 8$$

In Excel.

Integration
Options:

- Basic Rule
- Identity substitution/algebra (if a rational function has a numerator with a larger degree than the denominator)
- Substitution
- Change of Variable (substitution with an identity)