1/20/2022

2.2 Limits

The limit is the value that the function is approaching when x is getting closer to a particular number.

$$\lim_{x \to c} f(x) = L$$

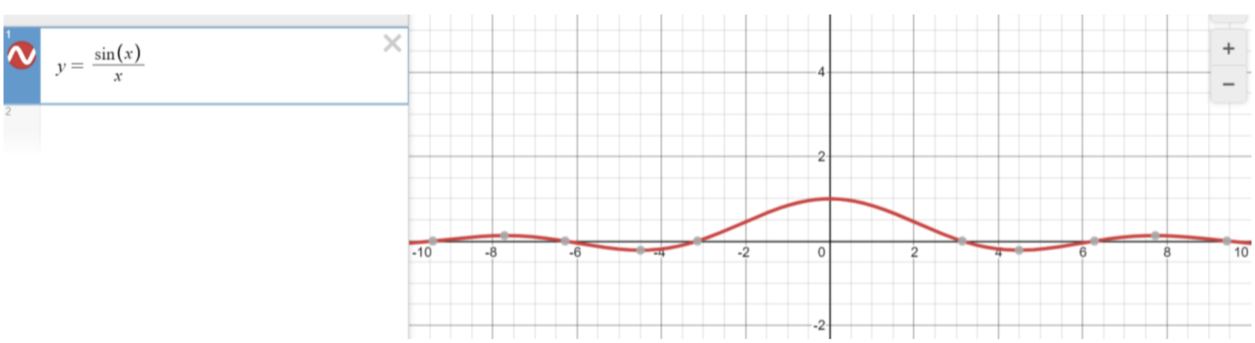
"The limit of f(x) as x approaches c is equal to L."

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$$

From Excel:

x	f(x)=(x^2-4)/(x-2)
3	5
2.5	4.5
2.1	4.1
2.01	4.01
2.001	4.001
2.0001	4.0001
2.000001	4.000001
1	3
1.9	3.9
1.99	3.99
1.999	3.999
1.9999	3.9999
1.99999999	4

What is the limit of $f(x) = \frac{\sin x}{x}$ as x goes to 0?

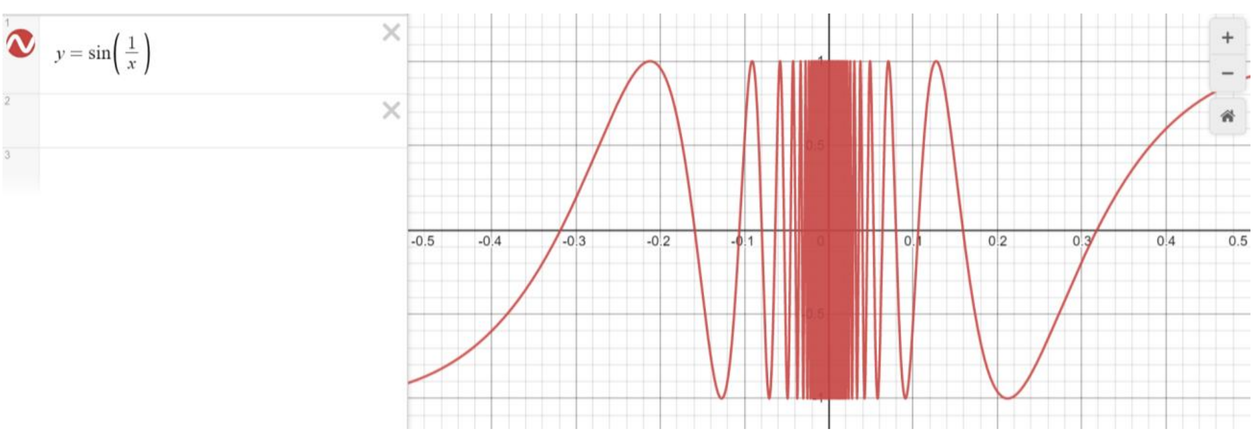


x	f(x)=sinx/x
1	0.841470985
0.5	0.958851077
0.1	0.998334166
0.01	0.999983333

0.001	0.999999833
0.0001	0.999999998
-1	0.841470985
-0.1	0.998334166
-0.01	0.999983333
-0.001	0.999999833

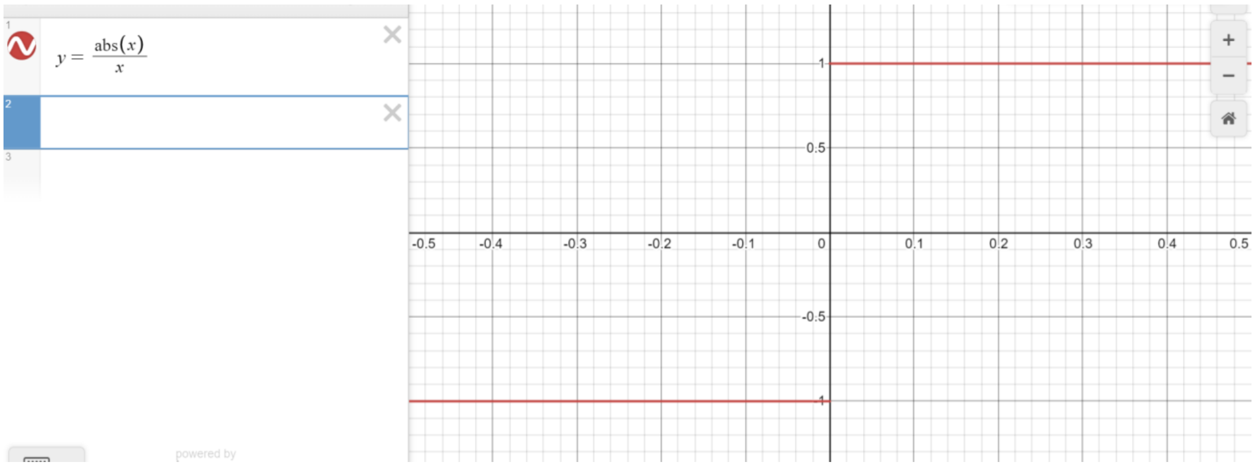
The limit of $\frac{\sin(x)}{x}$ is equal to 1 as x goes to 0.

x	f(x)=sin(1/x)
1	0.841470985
0.5	0.909297427
0.1	-0.54402111
0.01	-0.50636564
0.001	0.826879541
0.0001	-0.30561439
-1	-0.84147098
-0.1	0.544021111
-0.01	0.506365641
-0.001	-0.82687954
-0.0001	0.305614389
-0.00001	-0.0357488
-0.000001	0.349993502
0.000001	-0.3499935



This function does not have a limit. The values on either side of 0 are oscillating faster and faster and not approaching anything.

Another example: $f(x) = \frac{|x|}{x}$



This function has no limit at x=0, because the two sides near 0 do not approach the same value. The negative values go to -1, and the positive values got to 1.

Two-sided limits are the default. We want numbers that are bigger than the target and numbers smaller than the target to approach the same function value (y-value).

One-sided limits

Left-side limit

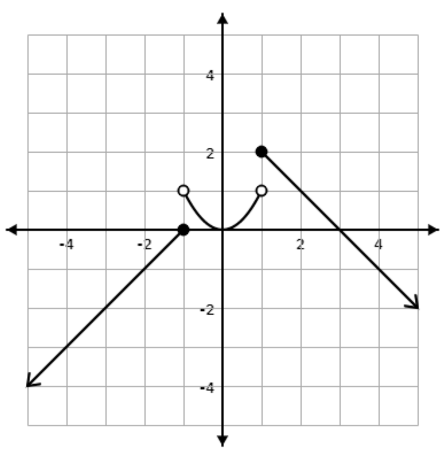
$$\lim_{x \to c^-} f(x) = L$$

Right-side limit

$$\lim_{x \to c^+} f(x) = L$$

One way of thinking about the two-sided limit is that it exists when the left-sided and right-sided limit agree (they are equal).

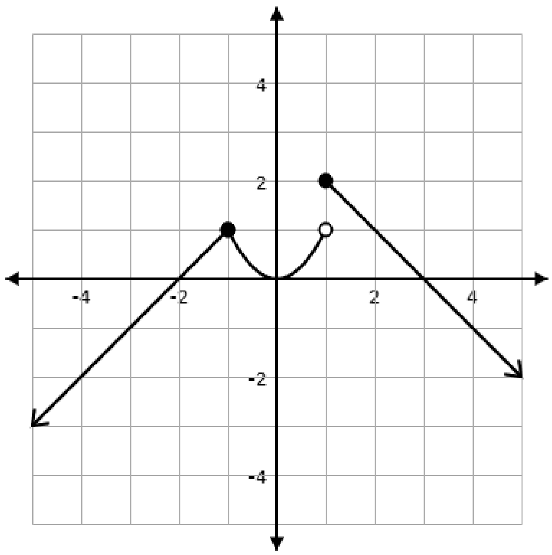
https://www.graphfree.com/



Using the graph of f(x) above, what is the limit of the function as x goes to 0? → 0

This section of the graph is smooth and connected, and the function is defined at 0, so the limit is the value of the function.

The limit does not exist at -1 (or 1) because the left- and right-sided limits are not the same value.



The left piece and the center piece in this graph have a limit (=1 at x=-1), but the right side and the middle piece do not have the same value at the endpoint x=1, and so the limit (two-sided limit) does not exist here.

2.3 Limit Laws

If you want to find the limit of a function f(x)=x, limit as x approaches c is equal to c.

When $f(c)$ is defined, then the limit of f(x) as x goes to c is $f(c)$ (when f(x) is "smooth")

The limit of a constant function is always the constant.

Limit of a function h(x) = f(x)+g(x)… then the limit of h(x) is equal to the limit of f(x) plus the limit of g(x).

Same for subtraction.
Same for multiplication if h(x)=f(x)g(x)
Same for division. Same for powers and roots.

$$\lim_{x\to 0}\sqrt{\frac{\sin(x)}{x}} = \sqrt{\lim_{x\to 0}\frac{\sin(x)}{x}} = \sqrt{1} = 1$$

What is the limit of $f(x) = \frac{x^2-4}{x-2}$ as x goes to 2?

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} x+2 = 4$$

$$\lim_{x\to 2} \sqrt[3]{\left(\frac{x^2-4}{x-2}\right)^7} = \sqrt[3]{4^7}$$