2/11/2021

Linear Regression (2.5) and Chapter 5 (5.1-5.3)

Line of Best Fit
Example 1: Data from Correlation and Regression Handout

$$y = 1.28x - 22.63$$

1.28 is the slope
-22.63 is the y-intercept

X were test scores from Test 1
Y were test scores from the Midterm

Slope is a rate where the midterm score is numerator and the denominator is the test score 1
If I change the Test 1 score by 1 unit, how much does the Midterm score change? = slope

For each one unit change of the Test score, the midterm score will go up by 1.28 points (an average)

For the intercept: not all intercepts can be interpreted in the real world
The y-value (Midterm score) when the x value (Test 1 score) is equal to 0.
We can't interpret this value here because no one can get a negative test score (smallest score is 0).

A restriction on the domain: x (test 1 score) has to be large enough so that Midterm is 0 or larger.

Ex from book (2.5):

$$y = 1.29x - 2473.89$$

y was energy usage in Quads (quadrillion BTUs)
x was year (time)

For every additional year (x), energy usage (y) increases by 1.29 Quads
Can we interpret the intercept? No such thing as negative energy usage

What is the energy in 2020, as predicted by our equation?

Chapter 5

Composition of Function

Review of Algebra of Functions

$$f(x), g(x)$$

Addition: $(f+g)(x) = f(x) + g(x)$
Subtract: $(f-g)(x) = f(x) - g(x)$
Multiply: $(fg)(x) = f(x)g(x)$
Divide: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Composition of Functions

$$(f \circ g)(x) = f\big(g(x)\big)$$

$\circ$ is read "composed of"
Replace x in the function f with the function g

$$f(x) = x^2 + 2x + 3, g(x) = x - 6$$

$$(f \circ g)(x) = f\big(g(x)\big) = (x - 6)^2 + 2(x - 6) + 3$$

$$(g \circ f)(x) = g\big(f(x)\big) = (x^2 + 2x + 3) - 6$$

$$(f \circ f)(x) = f\big(f(x)\big) = (x^2 + 2x + 3)^2 + 2(x^2 + 2x + 3) + 3$$

$$(g \circ g)(x) = g\big(g(x)\big) = (x - 6) - 6$$

Think of the outer function as the "template" and fill in or replace x with the inside function

Domain and range of composition of functions.

$$f(x) = x^2 + 1, g(x) = \sqrt{x + 2}$$

$$(f \circ g)(x) = \left(\sqrt{x + 2}\right)^2 + 1 = x + 2 + 1 = x + 3$$

$f\big(g(x)\big)$ evaluated at a value of x:

$$f\big(g(2)\big) = 5$$

First find $g(2)$. Plug value of 2 into g: $g(2) = 2$
Then plug 2 into f: plugging the result of g(2) into the function f

$$f\big(g(7)\big) = 10$$

First find $g(7)$. Plug value of 2 into g: $g(7) = 3$
Then plug 3 into f: plugging the result of g(7) into the function f

But what if I plug in $x = -4$?

$$f\big(g(-4)\big) = ?$$

To determine the domain, first look at the domain of $g(x)$, and also the domain of $f(x)$ separately first. If $g(x)$ is defined on all real numbers, then $f(g(x))$ has the same domain as $f(x)$. But, if $g(x)$ is restricted, then the domain of $f(g(x))$ is where both $f(x)$ and $g(x)$ are defined.

$$D_g: [-2, \infty)$$

This is also the domain of $f(g(x))$

One-to-one functions (injective): for every x there is only one y, AND the reverse is also true: for every y there is only one x.

For functions: vertical line test
For one-to-one functions: horizontal line test

Linear functions are always one-to-one (except for horizontal lines themselves)
Quadratic functions are not one-on-one (in order to make them one-to-one, we'd have to restrict the domain from the axis of symmetry)
Even functions are never one-to-one on their entire domains
Odd functions could be one-to-one, functions with no symmetry could be

To test for one-to-one-ness, the easiest way is to graph it.

We want this property because we are trying to find the inverses of functions that are themselves inverses.

If $g(x)$ is an inverse of $f(x)$, then $f\big(g(x)\big) = x$, $g\big(f(x)\big) = x$.
The inverse function notation for $f(x)$ is $f^{-1}(x)$. The (-1) is not a reciprocal. $f^{-1}(x) \neq \frac{1}{f(x)}$. Instead, read this notation as "f-inverse of x".

If we want $\frac{1}{f(x)}$, write $f^{(-1)}(x)$.

Inverse: $f\big(f^{-1}(x)\big) = x, f^{-1}\big(f(x)\big) = x$

Finding an inverse:

1) Switch x and y in your equation
2) Solve for the new y

$$f(x) = 2x + 3$$

Find the inverse of $f(x)$.

$$y = 2x + 3$$
$$x = 2y + 3$$
$$x - 3 = 2y$$
$$\frac{(x-3)}{2} = y$$
$$f^{-1}(x) = \frac{x-3}{2}$$

Are they really inverses? What is $f\big(f^{-1}(x)\big)$?

$$2x + 3$$
$$2\left(\frac{x-3}{2}\right) + 3$$

$$x - 3 + 3$$
$$x$$

$$\frac{x-3}{2}$$
$$\frac{(2x+3)-3}{2}$$
$$\frac{2x}{2}$$
$$x$$

With a rational function:

$$f(x) = \frac{4x-1}{x-3}$$

$$y = \frac{4x-1}{x-3}$$

Switched x and y.

$$x = \frac{4y-1}{y-3}$$

Solve for y.

$$x(y-3) = 4y-1$$
$$xy - 3x = 4y - 1$$
$$xy - 4y = 3x - 1$$
$$y(x-4) = 3x - 1$$
$$y = \frac{3x-1}{x-4}$$

$$f^{-1}(x) = \frac{3x-1}{x-4}$$

Ex.3

$$f(x) = \frac{2x+1}{x-6}$$

$$y = \frac{2x+1}{x-6}$$

Switch x and y

$$x = \frac{2y+1}{y-6}$$

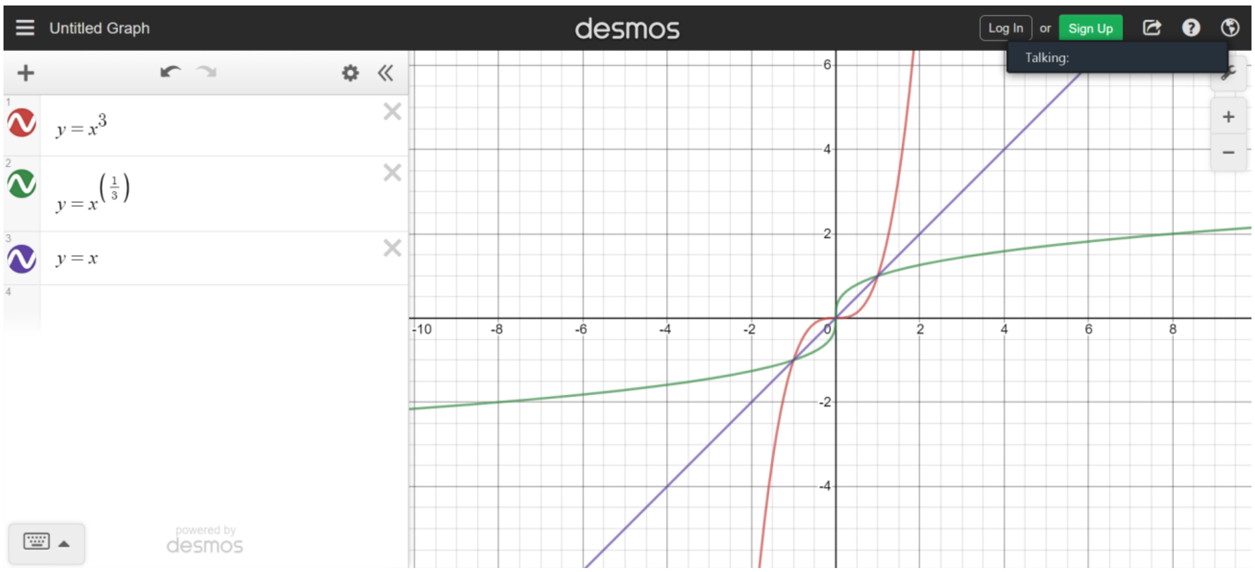
Solve for y.

$$x(y-6) = 2y+1$$
$$xy - 6x = 2y + 1$$
$$xy - 2y = 6x + 1$$
$$y(x-2) = 6x + 1$$

$$y = \frac{6x+1}{x-2}$$

$$f^{-1}(x) = \frac{6x+1}{x-2}$$

Symmetry between the function and it's inverse.



Symmetry is around the line $y = x$.

Test 1 content ends here.