1/28/2021

Chapter 1

Sets – ways of writing sets
A set is a collection of elements (you can think of it as a box with stuff in it).

1) Word – verbal description
2) List method used { }
3) Set builder notation $\{x|condition\}$

Verbal description: The set of all even positive integers.
List: $\{2, 4, 6, 8, \ldots\}$
Set builder: $\{x|x \text{ is even}, x \text{ is a positive integer}\}$

Sets of Numbers:

$\mathbb{N}$ = Set of Counting Numbers = $\{1, 2, 3, \ldots\}$ positive whole numbers
Whole Numbers = $\{\mathbf{0}, 1, 2, 3, \ldots\}$ non-negative whole numbers (W)
$\mathbb{Z}$ = Set of Integers = $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ positive and negative whole numbers
$\mathbb{Q}$ = Set of Rational Numbers = any number that can be expressed as the ratio of two integers = $\left\{\frac{p}{q}\middle|p, q \in \mathbb{Z}, q \neq 0\right\}$ (Q stands for quotient), terminating or repeating decimals.

I = set of Irrational Numbers = the set of numbers in the real set that are not rational
$\mathbb{R}$ = The set of Real Numbers = any number that can be used to measure something… includes all rational numbers and all irrational numbers, includes π, e, square roots, etc.; includes non-repeating, non-terminating decimals.

$\frac{1}{7} = 0.142857142857\overline{142857}$ …. rational

$3.101001000100001 \ldots$ irrational

$\mathbb{C}$ = set of Complex numbers = any number that can be written in the form $a + bi$ where $i = \sqrt{-1}$. Includes imaginary numbers (complex but have no real component).

Interval notation is used to express sets of real numbers

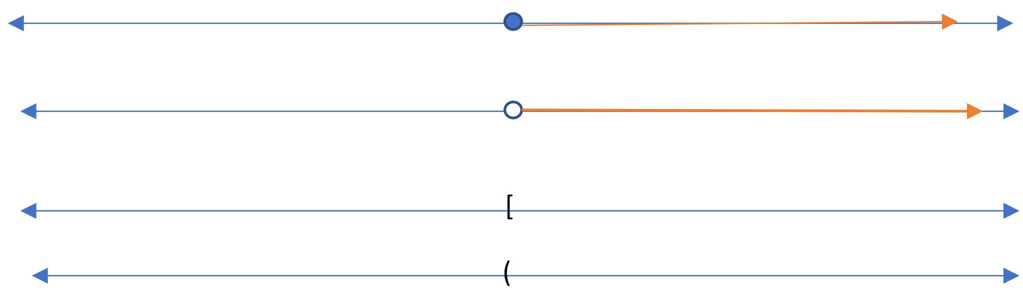
$$\mathbb{R} = (-\infty, \infty)$$

Interval notation: reports a starting value, and an ending value. What differs from one interval to another is whether the endpoints are included or not.

$(a, b) \sim (1,2)$ all the real numbers between 1 and 2, but not including the endpoints.
$[a, b] \sim [1,2]$ all the real numbers between 1 and 2, also including both endpoints.
$[1,2)$ includes 1, but not 2

$(1,2]$ includes 2, but not 1
∞ is not a number, and it always takes round brackets ( ).

Open circle means not included, (round parentheses), while a closed circle means that that endpoint is included, and use a square bracket



$$[4, \infty)$$

Vs.

$$(4, \infty)$$

What if I have two intervals and I need the numbers that are either in both intervals, or in either interval?

If we need to be in either interval, but there is no overlap, then $(a, b) \cup (c, d)$

Numbers less than -3 or bigger than 4: $(-\infty, -3) \cup (4, \infty)$

If you need to be in BOTH intervals or there is overlap, then you need to combine them into a single interval. It's easiest to figure out by drawing.

Numbers bigger than 3, but less than 5:

Simplify as much as possible!

Cartesian coordinates/Rectangular coordinates = represent points in a plane

Quadrant I – is all coordinates positive
Quadrant II is negative x and positive y (counterclockwise)
Quadrant III is both negative
Quadrant IV is positive x and negative y

Symmetry

y-axis symmetry: change the sign of the x-coordinate (x,y) becomes (-x,y)
x-axis symmetric: change the sign of the y-coordinate (x,y) becomes (x,-y)
origin symmetry: change the sign of both coordinates (x,y) becomes (-x,-y)

Equation to test symmetry:

$$y = x^2$$

To test for y-axis symmetry: replace x with -x in the equation (if the result is the same as the original equation, then we have y-axis symmetry).

$$y = (-x)^2 = x^2$$

Yes, Since negatives cancel in pairs.

To test for x-axis symmetry, replace y with -y

$$-y = x^2$$

No, because the negative can't cancel

To test for origin symmetry, replace x with -x AND y with -y

$$-y = (-x)^2$$

No, because there are three negatives, and so one is leftover.

In general, equations can have 1) only y-axis symmetry, 2) only x-axis symmetry, 3) only origin symmetry, 4) all three symmetries, 5) none

Distance formula and the midpoint formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Suppose we have the points (-2,1) an (-4,3). What is the distance between the points?

$$(-2,1) = (x_1, y_1), (-4,3) = (x_2, y_2)$$

$$d = \sqrt{\big(-4 - (-2)\big)^2 + (3 - 1)^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

What is the midpoint between these two points?

$$\left(\frac{-2 + (-4)}{2}, \frac{1 + 3}{2}\right) = \left(-\frac{6}{2}, \frac{4}{2}\right) = (-3,2)$$

Suppose that these two points were the endpoints of the diameter of a circle.

$$(x-h)^2+(y-k)^2=r^2$$

r is the radius, and (h,k) is the center.
The distance between the two endpoints of the diameter is twice the size of the radius. $2r=d$

$$2\sqrt{2}=d=2r$$
$$r=\sqrt{2}$$

Center is the midpoint of the diameter.

$$\left(x-(-3)\right)^2+(y-2)^2=\left(\sqrt{2}\right)^2$$

$$(x+3)^2+(y-2)^2=2$$

Relations (and Functions)

An equation (or other mathematical relationship) between two values/variables (inputs and outputs)

$$y=x^2, x^2+y^2=1, x=y^3+y$$

Can also be done in tables, or lists of pairs of points.

$$R=\{(blue,tall),(green,dark),(brown,young)\}$$

Relations have domains (set of “input” values), and ranges (set of “output” values), and inverses

Domain of R is {blue, green, brown}
Range of R is {tall, dark, young}

Equations of Lines

Standard form of a line: $Ax+By=C$ (general form $Ax+By+C=0$)
Y-intercept form: $y=mx+b$
Point-Slope form of the line: $y-y_1=m(x-x_1)$
Horizontal line: $y=b$
Vertical line: $x=a$

Slope: $m=\frac{y_2-y_1}{x_2-x_1}$

Suppose we want to find the equation of the line passing through (-2,1) an (-4,3).

$$m=\frac{3-1}{-4-(-2)}=\frac{2}{-2}=-1$$

$y-1=-1(x-(-2))$ (point-slope)
$y-1=-1(x+2)$

$y - 1 = -x - 2$
$y = -x - 1$ (y-intercept)
$x + y = -1$ (standard)

Intercepts : are the points on the graph that cross one axis

x-intercept crosses the x-axis (y=0) $(a, 0)$ plug in 0 for y, and then solve for x to get a value
y-intercept crosses the y-axis (x=0) $(0, b)$ plug in 0 for x, and the solve for y to get b value

If the intercept is the origin (0,0), then you have to find a second point to plot for the line

Functions

Function are relations that have the property that for each input value (x) there is only one output value (y)

$$R = \{(1,2), (1,4), (2,5), (3,7), (4,9)\}$$

Not a function: if I choose x=1, I have two options for y: 2, or 4

$$S = \{(1,3), (2,4), (3,7), (4,7), (5,1)\}$$

Function: if I choose any x, I know what y has to be

Graphical version of the function test: Vertical line test

If a vertical line crosses the graph of relation more than once, then the relation is not a function.

If you can solve for y without any $\pm$, then you have a function

Domain and range of functions

Domains are usually easier to figure out. If you solve for y first. Look for any places where the equation is not defined.

$$y = x^2 + 2x - 3$$

Domain is all real numbers.

$$y = \sqrt{x}$$

Domain not all real numbers, domain is $x \geq 0$ or $[0, \infty)$. Range is also $y \geq 0, or\ [0, \infty)$.

Ranges are harder to figure out. Sometimes we have to graph the equation. Ranges are specific to the kind of function they are. We need inverses to find the range.

Function notation

$$y = mx + b$$
$$f(x) = mx + b$$

“f of x”

$$f(t) = m\boldsymbol{t} + b$$

Using function notation:

$$f(x) = x^2 + 2x - 3$$

$f(-1) = (-1)^2 + 2(-1) - 3 = -4 : (-1, -4)$
$f(0) = 0^2 + 2(0) - 3 = -3 : (0, -3)$
This direction gives you the x coordinate, and you find the y-coordinate

$$f(x + h) = (x + h)^2 + 2(x + h) - 3$$

$$f(x) = 0$$

This direction gives you the y-coordinate, and you need to find the x-coordinate.

$$0 = x^2 + 2x - 3$$
$$0 = (x - 1)(x + 3)$$
$$x = 1, x = -3$$

Two points on the graph: $(1,0), (-3,0)$

Difference quotient:

$$\frac{f(x + h) - f(x)}{h}$$

$$f(x) = x^2 + 2x - 3$$

$$f(x + h) = (x + h)^2 + 2(x + h) - 3$$
$$(x + h)^2 = (x + h)(x + h) = x^2 + 2xh + h^2$$

$$\frac{\left((x + h)^2 + 2(x + h) - 3 - (x^2 + 2x - 3)\right)}{h} = \frac{x^2 + 2xh + h^2 + 2x + 2h - 3 - x^2 - 2x + 3}{h}$$

$$\frac{2xh + h^2 + 2h}{h} = \frac{h(2x + h + 2)}{h} = 2x + h + 2$$

If there is a square root in the problem, you need to rationalize the numerator
If there is a rational equation in the problem, find a common denominator (in the numerator), can pull out the divide by h as (1/h) multiplier

Arithmetic of functions: $(f + g)(x) = f(x) + g(x), (f - g)(x) = f(x) - g(x), (fg)(x) = f(x)g(x), \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$f(x) = x^2 + 2x, g(x) = x + 1$$

$$(f + g)(x) = x^2 + 2x + x + 1 = x^2 + 3x + 1$$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x}{x + 1}$$

Graphing functions: plot several points and connect the dots.

Properties:
Increasing or decreasing

Increasing: goes up to the right (as x increases, y also increases)
Decreasing: go down to the right (as x increases, y decreases)
Defined as intervals in x

Minimum values (minima) are places on the graph where all points nearby are higher than the value
Maximum values (maxima) are places on the graph where all nearby points are lower than the value

Constant intervals: where the graph is horizontal