MAT 161, Final Exam, Spring 2021 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. Solve the system $\begin{cases} 5x + 12y + z = 10 \\ 2x + 5y + 2z = -1 \\ x + 2y - 3z = 5 \end{cases}$ by any method. (12 points)

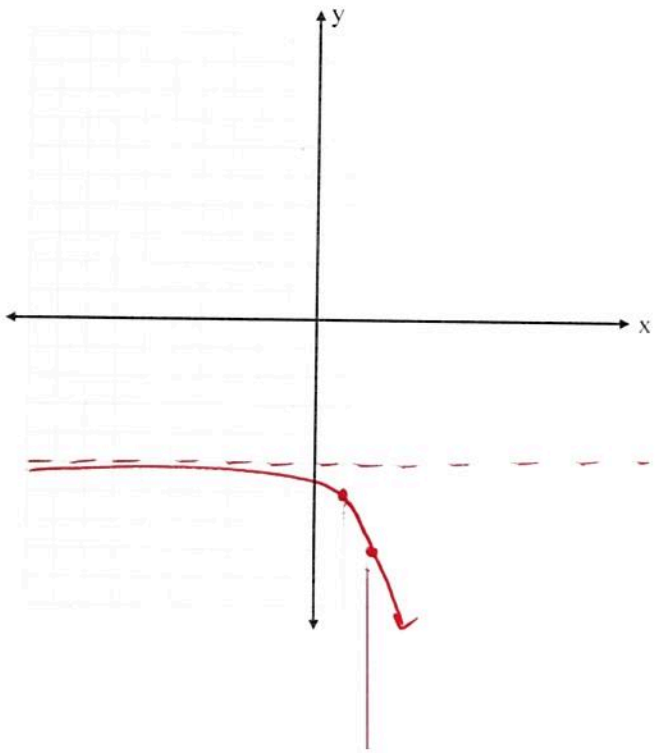
$$\left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

System is inconsistent

2. Sketch the graph of the curve $f(x) = -3^{x-1} - 5$. State the domain and range. (10 points)

D: $(-\infty, \infty)$

R: $(-\infty, -5)$



3. Find the inverse function of $f(x) = 10e^x + 7$. State the domain and range of the inverse. (8 points)

$f(x)$ $\quad f^{-1}(x)$

D: $(-\infty, \infty)$ $\quad (7, \infty)$

R: $(7, \infty)$ $\quad (-\infty, \infty)$

$x = 10e^y + 7$

$\frac{x-7}{10} = e^y$

$\ln\left(\frac{x-7}{10}\right) = y = f^{-1}(x)$

4. Solve the following equations without using a calculator. (8 points each)

a. $\log(x+4) - \log 2 = \log(5x+1)$

$\log\left(\frac{x+4}{2}\right) = \log(5x+1)$

$x+4 = 10x+2$

$2 = 9x$

$x = 2/9$

b. $e^{2x} - 4e^x - 12 = 0$ $\quad u = e^x$

$u^2 - 4u - 12 = 0$

$(u-6)(u+2) = 0$

$u = 6, -2$

$u = -2 \rightarrow e^x = -2$ no solution

$e^x = 6$

$x = \ln 6$

5. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -2x^2 + 3x - 5$. (10 points)

$$\frac{-2(x+h)^2 + 3(x+h) - 5 - (-2x^2+3x-5)}{h} =$$

$$\frac{-2x^2 - 4xh - 2h^2 + 3x + 3h - 5 + 2x^2 - 3x + 5}{h} =$$

$$\frac{-4xh - 2h^2 + 3h}{h} = \frac{h(-4x-2h+3)}{h} = -4x - 2h + 3$$

6. Find an equation of the line with the following properties passing through the points $(2, -3)$ and $(4, -3)$ in rectangular coordinates. (10 points)

$m = \frac{-3-(-3)}{2-4} = \frac{0}{-2} = 0$

$y = -3$

7. If $f(x) = |x|$, write the function that has all the following transformations applied in order: (8 points)
   a. Shift left 4 units
   b. Reflect over the x-axis
   c. Compress by a factor of 2
   d. Shift up by 3

$g(x) = |x-4|$

$h(x) = -|x-4|$

$j(x) = -\frac{1}{2}|x-4|$

$k(x) = -\frac{1}{2}|x-4| + 3$

8. Given $g(x) = \sqrt{x-4}$, $h(x) = x + \frac{1}{x}$, find $(h \circ g)(x)$ and state the domain. (8 points)

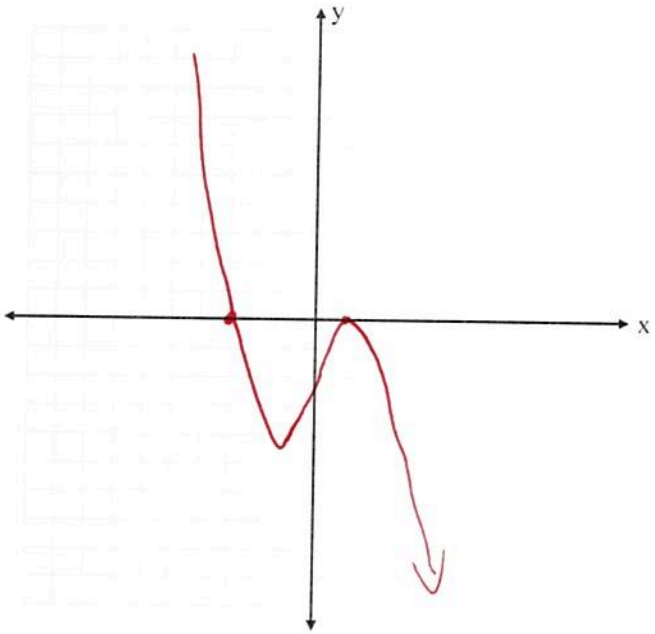
$= \sqrt{x-4} + \frac{1}{\sqrt{x-4}}$

$x \geq 4$

D: $(4, \infty)$

9. Find all the possible rational zeros of the polynomial $f(x) = -x^3 - x^2 + 5x - 3$. Use them to factor the polynomial, and find all the real (and complex, if any) zeros. Write the polynomial in factored form. Sketch the graph. (10 points)

$-(x+3)(x-1)^2$



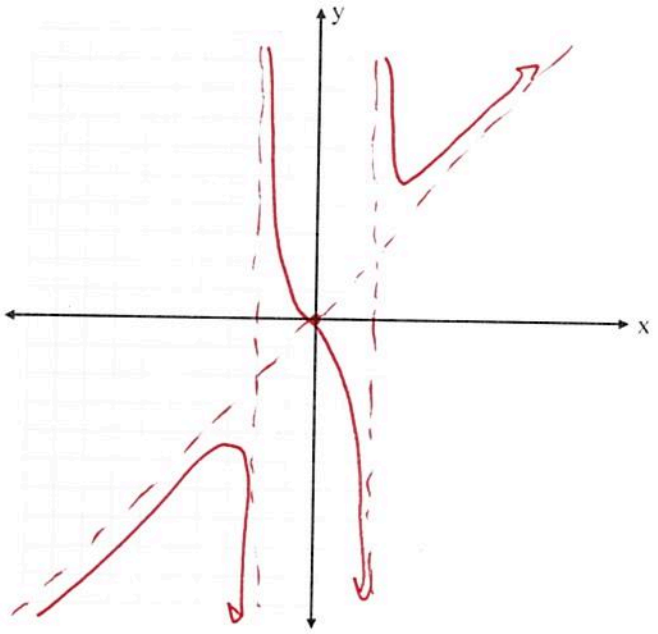
10. Sketch the graph of the function $f(x) = \frac{x^3+x}{x^2-4}$, but finding i) any intercepts, ii) any vertical asymptotes or holes, iii) any horizontal or slant asymptotes. (10 points)

$\frac{x(x^2+1)}{(x-2)(x+2)}$

VA: $x=2, x=-2$

int: $x=0, y=0$

SA: $y=x$



11. Find all zeros of $P(x) = x^3 - 4x^2 - 11x + 30$. (6 points)

$(x+3)(x-2)(x-5)$

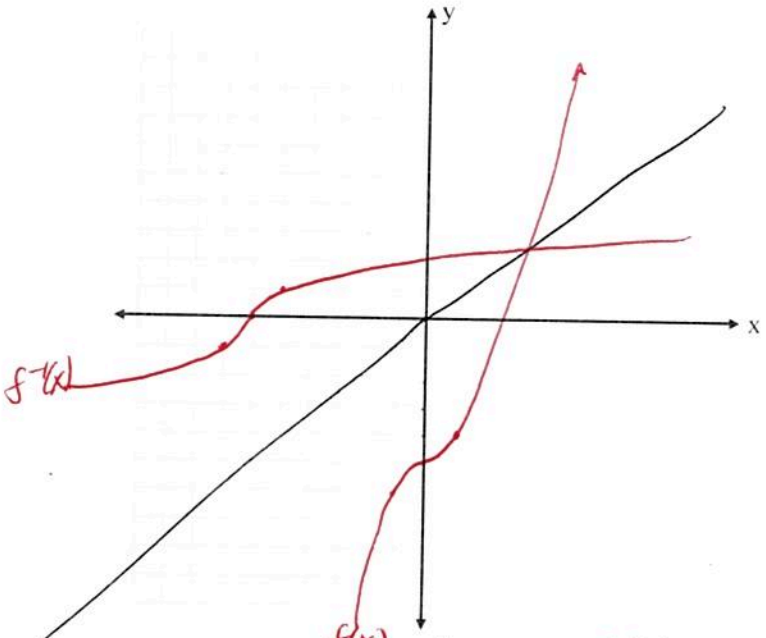
$x = -3, 2, 5$

12. Find $f^{-1}(x)$, if it exists for the following functions. Plot each inverse and the original function on the same graph.

a. $f(x) = x^3 - 5$

$x = y^3 - 5$

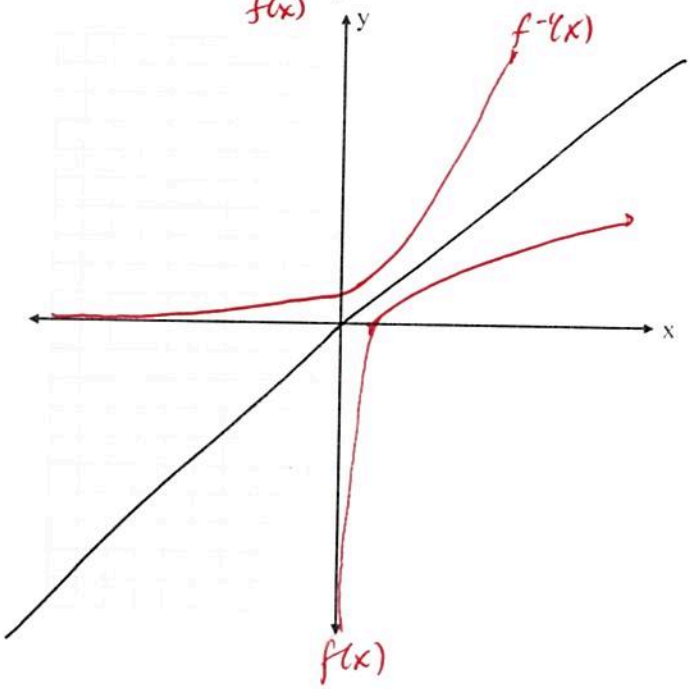
$\sqrt[3]{x+5} = y = f^{-1}$



b. $f(x) = \log_2 x$

$x = \log_2 y$

$2^x = y = f^{-1}$



13. Let $f(x) = 2x, g(x) = (x-1)^2$. Find $(f \circ g)(x)$. (5 points)

$= 2(x-1)^2$

14. Solve the following system: $\begin{cases} 2x - 3y + 2x = 7 \\ x + 4y \quad\quad = -1 \\ -x + y - 3x = -24 \end{cases}$. (10 points)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 10 \end{array}\right]$$

$X = -5,\ y = 1,\ z = 10$

15. Solve the following equations and inequalities. (5 points each)

a. $3^{3x} = 9^{x+1}$

$3^{3x} = 3^{2(x+1)}$

$3x = 2x + 2$

$x = 2$

b. $\log_2 x + \log_2(5x+5) = \log_2(10)$

$\log_2[x(5x+5)] = \log_2 10$

$5x^2 + 5x - 10 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

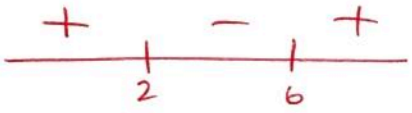
~~$x = -2$~~ $\log_2(-2)$ not defined

$x = 1$

c. $x^2 + 12 \geq 8x$

$x^2 - 8x + 12 \geq 0$

$(x-6)(x-2) \geq 0$



$(-\infty, 2] \cup [6, \infty)$

16. Find the partial fraction decomposition for $\frac{2x^2-7x+11}{x(x-1)^2(x^2+4)}$. Set up, but you do not need to solve for the constants.

$$\frac{A}{X} + \frac{B}{X-1} + \frac{C}{(X-1)^2} + \frac{DX+E}{X^2+4}$$

Do a COMPLETE ANALYSIS of the following functions and sketch the graph of each.

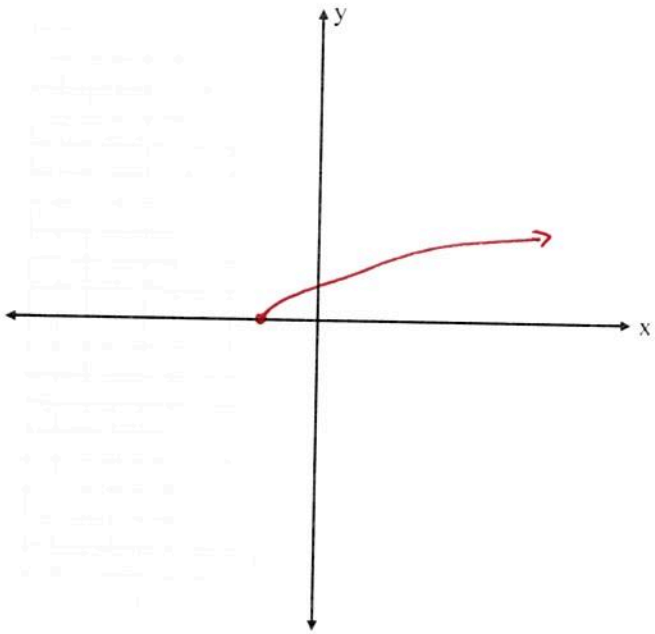
17. $f(x) = \sqrt{x+2}$ (10 points)

Domain: $X \geq -2$

X-Intercepts: $(-2, 0)$

Y-Intercept: $(0, \sqrt{2})$

Range: $[0, \infty)$



18. $g(x) = 7^{-x}$ (10 points)

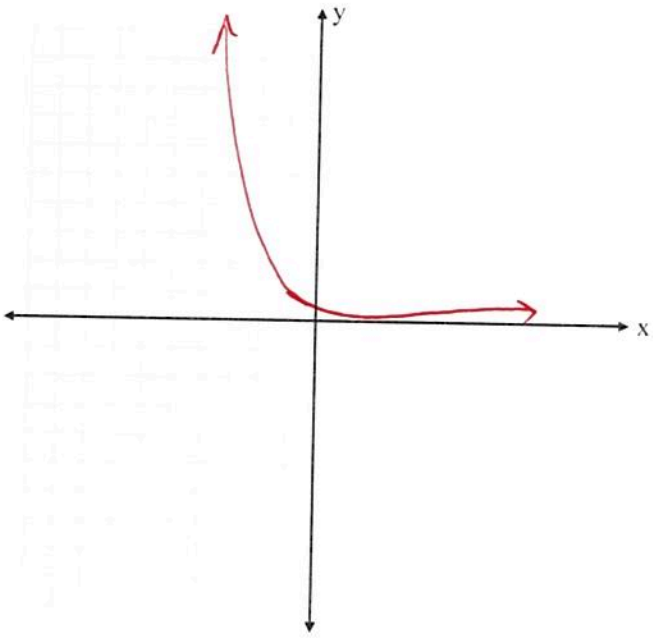
Domain: $(-\infty, \infty)$

Zeros: none

Y-Intercept: $(0, 1/7)$

Asymptote: $y=0$

Range: $(0, \infty)$



19. $y = 3 - 2x - x^2$ (10 points)

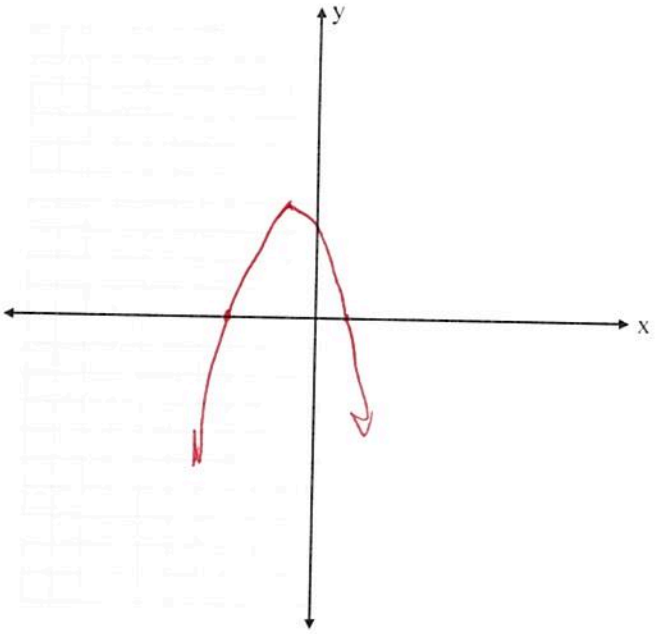
Domain: $(-\infty, \infty)$

Vertex: $(-1, 4)$

Y-Intercept: $(0, 3)$

X-Intercepts: $(-3, 0), (1, 0)$

Range: $(-\infty, 4]$



20. $f(x) = \ln(x-2)$ (10 points)

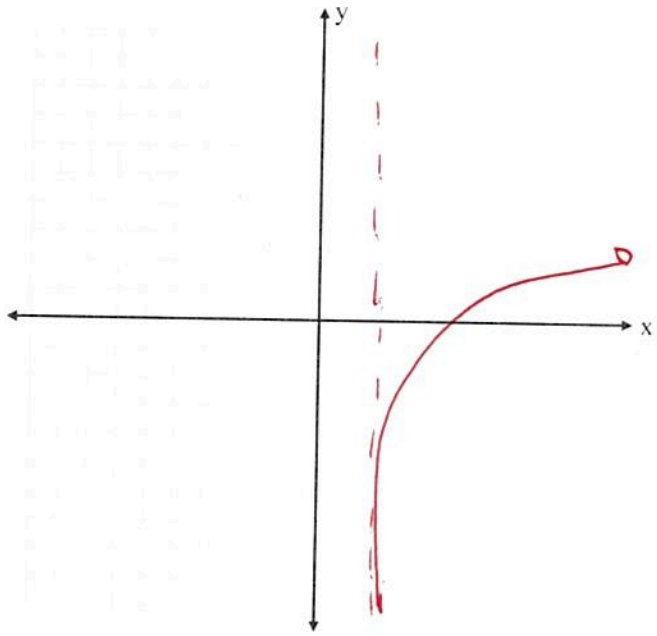
Domain: $(2, \infty)$

Y-Intercept: none

X-Intercpet: $(3, 0)$

Asymptote: $x = 2$

Range: $(-\infty, \infty)$



21. $y = \frac{2x-4}{x-2}$ (10 points)

Domain: $x \neq 2$

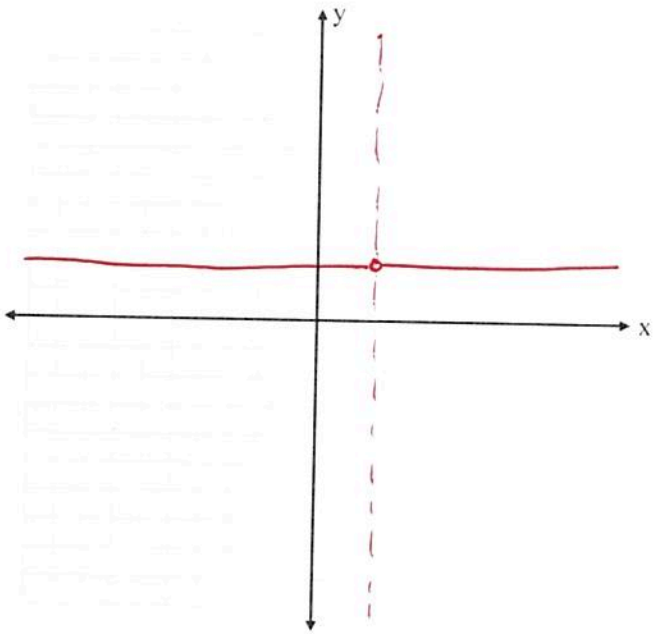
Y-Intercept: $(0, 2)$

X-Intercept: none

VA: none

HA: 2

$\frac{2(x-2)}{x-2} = 2$



22. Solve one of the two-dimensional systems and one of the three-dimensional systems below by the method of your choice, and characterize each as consistent or inconsistent, and if applicable, dependent or independent. You may do the others for bonus points. (7 points each)

a. $\begin{cases} 3x - 2y = -5 \\ 4x + y = 8 \end{cases}$

$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \end{array}\right]$

$X = 1, Y = 4$

Consistent, independent

b. $\begin{cases} x + 3y + 5z = 20 \\ y - 4z = -16 \\ 3x - 2y + 9z = 36 \end{cases}$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$X = 0, Y = 0, Z = 4$

Consistent, independent

c. $\begin{cases} 3x - y = 10 \\ 2x + 5y = 1 \end{cases}$

$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \end{array}\right]$

$X = 3, Y = -1$

Consistent, independent

d. $\begin{cases} x - 2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$x = 2, y = -1, z = 1.$

consistent, independent