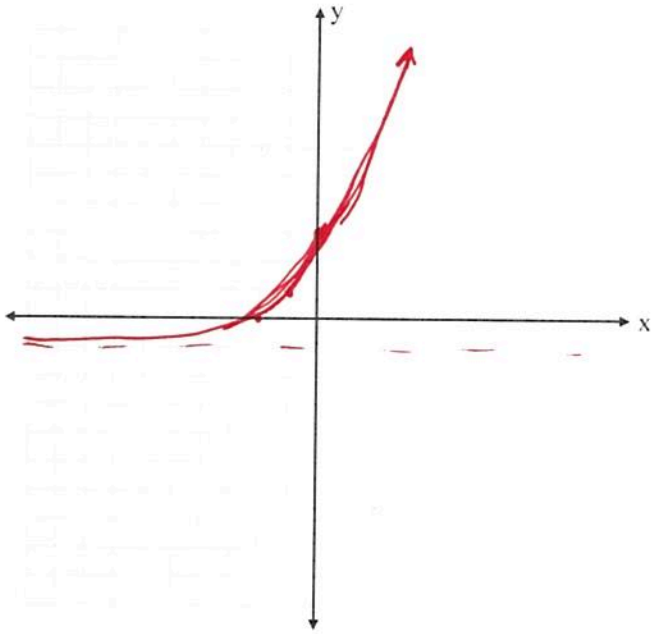
MTH 161, Exam #3, Spring 2021 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing"; partial credit will not be possible.

1. Sketch the graph of the curve $f(x) = 2^{x+2} - 1$. State the domain and range. (10 points)

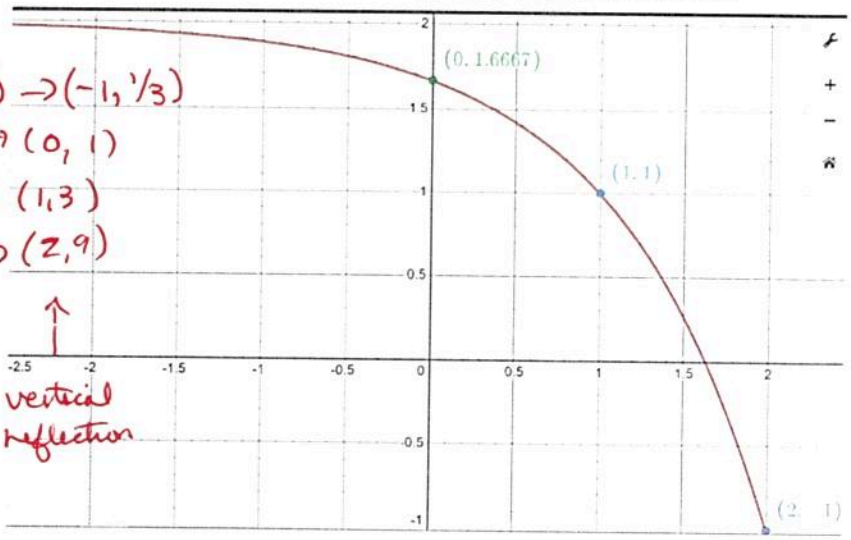
(0, 3)
(-1, 1)
(-2, 0)



D: $(-\infty, \infty)$

R: $(-1, \infty)$

2. For the graph shown below, write the equation of the function. Three key points are plotted. (6 points)



$(0, 5/3) \to (0, -1/3) \to (-1, -1/3) \to (-1, 1/3)$

$(1, 1) \to (1, -1) \to (0, -1) \to (0, 1)$

$(2, -1) \to (2, -3) \to (1, -3) \to (1, 3)$

$(3, -7) \to (3, -9) \to (2, -9) \to (2, 9)$

↑ vertical shift up 2

↑ horizontal shift 1

↑ vertical reflection

$f(x) = -3^{x-1} + 2$

3. Suppose that you have $6000 to invest. How much would you have after 6 years earning 5.4% compounded semi-monthly? (12 points)

$$6000\left(1+\frac{.054}{24}\right)^{144} = \$8292.87$$

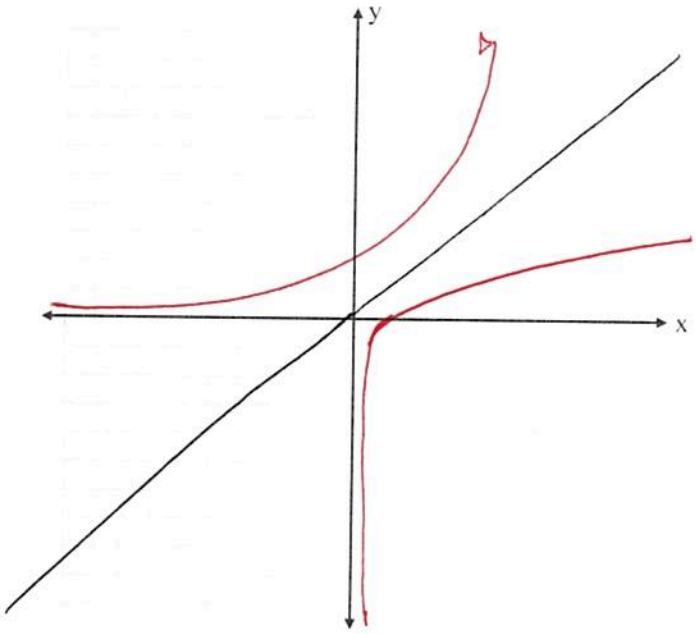
4. Find the inverse function of $f(x) = 2e^x + 1$. Sketch the graph and its inverse on the same axis. (12 points)

$$y = 2e^x + 1$$

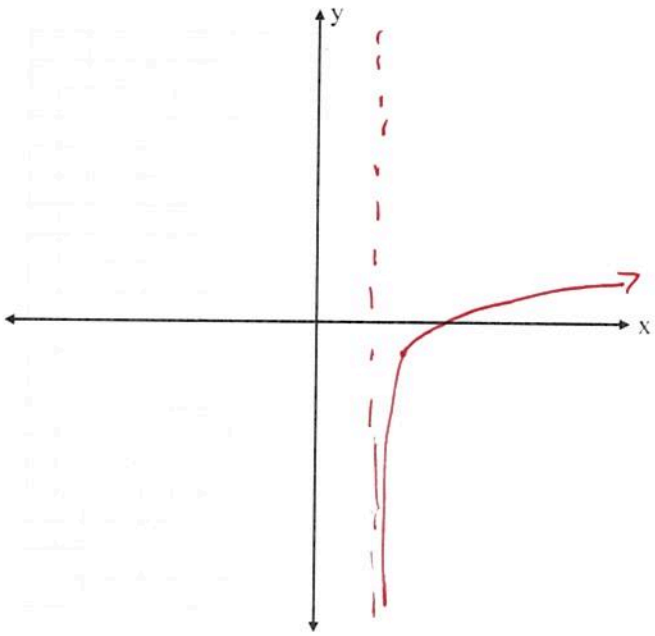
$$x = 2e^y + 1$$

$$\frac{x-1}{2} = e^y$$

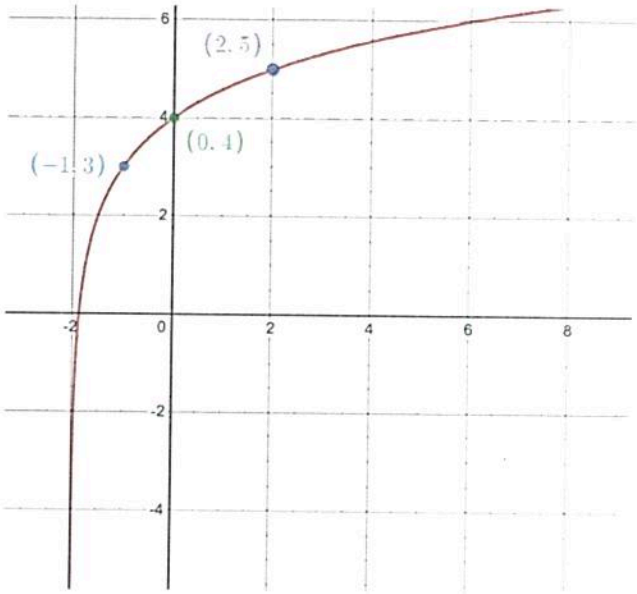
$$\ln\left(\frac{x-1}{2}\right) = y = f^{-1}(x)$$



5. Find the domain and range of the graph $f(x) = \log(x - 2) - 1$. Sketch the graph. (6 points)



6. The graph of a logarithmic function is shown below. Find the equation of the function if the base of the log is an integer. (16 points)



$(-1,3) \rightarrow (1,3) \rightarrow (1,0)$

$(0,4) \rightarrow (2,4) \rightarrow (2,1)$

$(2,5) \rightarrow (4,5) \rightarrow (4,2)$

$(-3/2,2) \rightarrow (1/2,2) \rightarrow (1/2,-1)$

↓ horizontal shift 2

↓ vertical shift 3

$f(x) = \log_2 (x+2) + 3$

7. Expand the logarithmic function $\log\left[\frac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$ into simpler logs and simplify expressions where possible. (10 points)

$\log(10x^2) + \log(\sqrt[3]{1-x}) - \log(7) - \log(x+1)^2$

$\log(10) + 2\log x + \frac{1}{3}\log(1-x) - \log 7 - 2\log(x+1)$

8. Write the expanded logarithm $\frac{2}{3}[2\ln(x+5) - \ln x - \ln(x^2-4)]$ as a single logarithmic expression. (10 points)

$\frac{2}{3}[\ln(x+5)^2 - \ln x - \ln(x^2-4)] =$

$\frac{2}{3}\left[\ln\left(\frac{(x+5)^2}{x(x^2-4)}\right)\right] = \ln\sqrt[3]{\left(\frac{(x+5)^2}{x(x^2-4)}\right)^2}$

$\ln\sqrt[3]{\frac{(x+5)^4}{x^2(x^2-4)^2}}$

9. Solve the following equations without using a calculator. (10 points each)

a. $9^x = \frac{1}{\sqrt[3]{3}}$

$3^{2x} = 3^{-1/3} \rightarrow 2x = -1/3$

$x = -1/6$

b. $\log(x+4) - \log 2 = \log(5x+1)$

$\log\left[\frac{(x+4)}{2}\right] = \log(5x+1)$

$x+4 = 10x+2$

$2 = 9x$

$x = \frac{2}{9}$

c. $e^{4x} - 3e^{2x} - 18 = 0$

$e^{2x} = u$

$u^2 - 3u - 18 = 0$

$(u-6)(u+3) = 0$

$u = 6, \; u = -3$

$e^{2x} = -3$ no solution

$e^{2x} = 6$

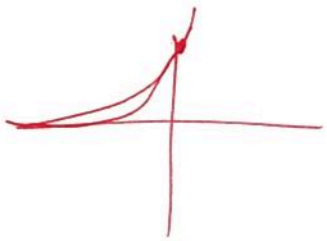
$2x = \ln 6$

$x = \frac{\ln 6}{2}$

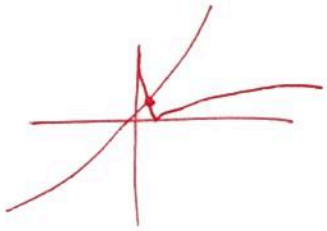
10. Solve the following equations. You may use a calculator (show any graphs used). Round answers to 4 decimal places. (7 points)

a. $7^{2x+1} = 3^{x+2}$

$X = .0899734 \approx 0.0900$



b. $2|\ln x| = 2^{x-1} + x$



$X = 0.53338$

≈ 0.5334

11. A bird species is in danger of extinction, with a population that is decreasing exponentially. Five years ago, the population was 14,000 and today only 1000 of the birds are alive. When the population drops below 100, the situation will be irreversible. Find the equation that models the birds' population, and determine when it reaches the point of no return. You may let "five years ago" be $t = 0$. How many years from now will it reach the point of no return? (10 points)

$t=0 \quad P = 14{,}000$

$t=5 \quad P = 1000$

$t=? \quad P = 100$

$P(t) = 14{,}000\, e^{kt}$

$P(5) = 1000 = 14{,}000\, e^{5k}$

$\frac{1}{14} = e^{5k}$

$\ln\left(\frac{1}{14}\right) = 5k$

$\frac{1}{5}\ln\left(\frac{1}{14}\right) = k \approx -0.5278$

$P(?) = 100$

$100 = 14{,}000\, e^{-0.5278t}$

$\frac{1}{140} = e^{-0.5278t}$

$-4.9416 = -0.5278t$

$t = 9.3625$

9.36 years.