3/6/2021

Scaling and Unit Conversion
The factors stated are in length units
We need to take that into account when working with area or volume

Metric conversion
English to Metric conversion
Scaling with area (in reverse)

Suppose I have a volume of liquid that is 13 cm^3, and I need to convert to mm^3. 10 mm in each centimeter.

$$\frac{10\ mm}{1\ cm}$$

$$13\ cm^3\left(\frac{10\ mm}{1\ cm}\right)^3 = 13\ cm^3\left(\frac{1000\ mm^3}{1\ cm^3}\right) = 13{,}000\ mm^3$$

Area measured in 6 in^2 and I want to covert to cm^2. 2.54 cm in each inch

$$\frac{2.54\ cm}{1\ in}$$

$$6\ in^2\left(\frac{2.54\ cm}{1\ in}\right)^2 = 6\ in^2\left(\frac{6.4516\ cm^2}{1\ in^2}\right) = 38.7096\ cm^2$$

Same idea works when you use scaling factors: so 1/6 size model, this is a linear scaling factor, so if you are scaling up area you would scale by $\left(\frac{1}{6}\right)^2$, and the volume by $\left(\frac{1}{6}\right)^3$.

Suppose I have a volume of 1000 cm^3 and I want to reproduce the model using only 15 cm^3 of material. What is the scaling factor that I need to use?

$$\frac{15\ cm^3}{1000\ cm^3} = 0.015$$

$$x^3 = 0.015$$
$$x = \sqrt[3]{0.015} = 0.015^{\frac{1}{3}} = 0.24662\ldots$$

Scaling factor is roughly ¼.

Probability

Basic probability rules:

1. All probabilities are between 0 and 1

    a. 0 probability means that it never occurs
    b. A probability of 1 means that it is certain to occur
2. All probabilities (associated with a particular kind of event) must add to 1.

These rules must apply to all probabilities distributions.

Notation for probabilities.
An Event (with a random outcome) call it a name like A (capital letter)
The probability of Event A is written as $P(A)$

$P(Heads), P(H), P(x = 1)$

x	1	2	3	4	5	6
$P(x)$	0.1	0.2	0.3	0.1	0.15	0.15

Valid probability distribution
What is the probability of obtaining a 5? $P(x = 5)$? 0.15
What is the probability of obtaining a value less than 3? $P(x < 3)$? 0.1+0.2= 0.3
What is the probability of obtaining a number bigger than 6? $P(x > 6)$? 0

Individual events in the table are sometimes called “simple” events. And events that are composed of more than one “simple” event are called “compound” events.

What is the probability of not obtaining a 5? $P(x \neq 5)$? 0.1+0.2+0.3+0.1+0.15=0.85
Complement of $P(x = 5)$. $P(x \neq 5) = 1 - P(x = 5) = 1 - 0.15 = 0.85$

If the event is A, the complement is $P(A') = P(A^c) = P(\sim A)$

Intersections (AND) and Unions (OR)

Events that are mutually exclusive: both events can’t occur at the same time. $P(A\ and\ B) = 0$

Or events : what is the probability of getting 2 or 3? $P(x = 2\ OR\ x = 3) = P(x = 2) + P(x = 3)$

If the events are not mutually exclusive, then the formula becomes: $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

A is getting a H on coin flip, and event B is getting a 4 on die roll
For a fair coin, and a fair die, the probability of a H is ½ and the outcome 4 has a probability of 1/6 on a fair die

$$P(H\ or\ 4) = P(H) + P(4) - P(H\ and\ 4)$$

When the events are independent (the outcome of one event doesn’t affect the other), then $P(A\ and\ B) = P(A)P(B)$.

$$P(H\ or\ 4) = \frac{1}{2} + \frac{1}{6} - \frac{1}{2} \times \frac{1}{6} = \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$$

Counting Rules

Probabilities for events that are built from equally likely simple events (equiprobable events) are calculated from a simple idea: divide the number of simple events in the “events” by the total number of possible outcomes

$$\mathrm{P}(A) = \frac{the\ number\ of\ ways\ to\ obtain\ event\ A}{the\ total\ number\ of\ possible\ outcomes = size\ of\ the\ sample\ space}$$

Multiplication Rule:
Multiply the number of outcomes in each simple event by the number of outcomes in each other event in the set (cross product in set theory).

Suppose I go on a long vacation, and I take 3 pairs of pants, 5 tops, and 2 pairs of shoes. How many different outfits can I wear before repeating any one combination?

$$3 \times 5 \times 2 = 30$$

Suppose I have a bowl filled with marbles: 7 green, 11 red, 5 blue, 10 white and 3 yellow.
How many marbles are in the bowl: 36
What is the probability of selecting a red marble? 11/36
If I keep that marble, what is the probability of selecting a green marble next? 7/35
What is the probability of first selecting a red marble, and then a green marble? $\frac{11}{36} \times \frac{7}{35}$

What is the probability of picking two marbles from the bowl, one of which is red and one of which is green? It could red, then green, OR it could be green and then red…. So I would have to add those two outcomes together. $\frac{11}{36} \times \frac{7}{35} + \frac{7}{36} \times \frac{11}{35} = 2\left(\frac{11}{36} \times \frac{7}{35}\right)$

Permutations

The number ways to organize a list of distinct elements in order (without repetition)

A baseball team has 9 players on the field at any one time. Suppose my team has 14 players. How many different ways are there to field 9 players?

$$P(n,r) = P_{n,r} = P_r^n = nPr = \frac{n!}{(n-r)!}$$

! = factorial

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

1! =1
2! =2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040

$$14P9 = \frac{14!}{(14-9)!} = \frac{14!}{5!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6$$

726,485,760.

Combinations – where is also no repetition, and order doesn't matter

$$C(n,r) = C_{n,r} = C_r^n = nCr = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

A standard deck of cards: contains 52 cards, 13 types of cards (Ace, 2, 3, 4, 5, 6, 7, 8, 8, 10, J, Q, K), and 4 suits (hearts, clubs, diamonds, and spades).

J, Q, K = face cards

5-card poker hand: How many different poker hands of 5 cards are there?

$$\binom{52}{5} = \frac{52!}{5!\,(52-5)!} = \frac{52!}{5!\,(47)!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 52 \times 51 \times 10 \times 49 \times 2$$

2,598,260 (size of the sample space for all 5-card poker hands)

What is the probability of obtaining a 3-of-a-kind in a 5-card poker hand?

What is the probability of obtaining 3 kings in a 5-card poker hand?

How many ways can we get three kings?

$$\binom{4}{3}\binom{48}{2} = 4 \times 1128 = 4512$$

$$\frac{4512}{2{,}598{,}260} = \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}}$$

$$\frac{13 \times 4512}{2{,}598{,}260} = \frac{13\binom{4}{3}\binom{48}{2}}{\binom{52}{5}}$$

What is the probability of obtain a 3-of-a-kind with face cards in a 5-card poker hand?

$$\frac{3 \times 4512}{2{,}598{,}260} = \frac{3\binom{4}{3}\binom{48}{2}}{\binom{52}{5}}$$

Conditional Probability

$P(A|B)$ = The probability of A given B: I the event B is happening already, and what is the probability of A knowing that?

If A and B are independent, then $P(A|B) = P(A)$

Many events may be dependent: If I know the gender of a person, does that effect the likelihood that that person will own power tools?

Formula for $P(A \text{ and } B) = P(A|B) \times P(B)$

Conditional probability: change the denominator