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Scaling and Unit Conversion

Scaling factors and unit conversion factors are usually only given in terms of one dimension: length (exception is when the base unit is a unit of area or volume: liter is a volume measure, but unit conversion are based on the volume unit, not the length unit; likewise for acreage in square yards: the conversion is based on area, not only length).

Suppose I want to find the area of an object that is 7' by 12'.   ' = feet, " = inches
Area = 7x12= 84 square-feet

Suppose I want to convert this to square-inches?
7' = 7x12 inches = 84 inches
12' = 12x12 inches = 144 inches
Area: 84x144 = 12,096 square inches.

12,096 square-inches/84 square-feet = 144 = 12x12

I know the area is 10 square-feet. What is that in square inches? 10x144 = 1440

Use one conversion factor for each dimension:
Length to length = use one multiple of the factor (feet to inches = x12)
Area to area = use two multiples of the factor (square-feet to square-inches = x12x12 =x144)
Volume to volume = use three multiples of the factor (cubic-feet to cubic-inches = x12x12x12 = x1728)

Conversion ratios from feet to inches = $\frac{12\ inches}{1\ foot}$

$$7\ feet\ \times \frac{12\ inches}{1\ foot} = 84\ inches$$

$$84\ ft^2\ \times \left(\frac{12\ inches}{1\ foot}\right)^2 = 12{,}096\ in^2$$

1 inch = 2.54 centimeters

10 cubic-inches to cubic centimeters

$$10\ in^3 \times \left(\frac{2.54\ cm}{1\ in}\right)^3 = 10 \times 2.54^3 cm^3 = 163.87\ cm^3$$

Suppose you are making a model of a house at ¼ scale. If the scale model used 11 cubic yards of materials, how much is needed for the full-scale house?

$$11\ \times 4^3 = 704\ yd^3$$

I have model of plane that uses 10 cubic feet of material, and the full-scale plane uses 1035 cubic feet of material. What is the scaling factor for the scale model?

$$\frac{1035}{10} = 103.5$$

This is scale of the volume but not the scale of the lengths?

For a volume, we need to take the cube root of the ratio: $\sqrt[3]{103.5} = 103.5^{\frac{1}{3}} \approx 4.7$
The scaling factor is $\frac{1}{4.7}$.

For area: we'd take the square root of the ratio of the areas.

Probability

A measure of the likelihood or chance that some random event is going to happen.

All probabilities are between 0 and 1 (inclusive).
An event with probability 0 is impossible
An event with probability 1 is certain (must happen)
Given all the events that can happen: their probabilities must add to 1.

Probability distribution: a list (or a table), or a formula that describes the probabilities for different outcomes, the sum of the all the probabilities in the distribution must add to 1.

Outcome of a coin flip	H	T
Probability	0.5	0.5

Outcome of a standard die	1	2	3	4	5	6
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Any change in the probability for one outcome has to offset by changes in other outcomes to compensate so that the probabilities always add to 1.

The mean outcome comes from a weighted average.

Suppose I have an event A. The probability of event A is written as $P(A)$.
If event A is rolling a 2 on a standard die, we could write $P(X = 2)$.

In probability X (capital letters) refer to the random variable, and x (lower case letters) refer to the outcome and their values.

When an event contains a single outcome in the set of all outcomes (sample space), it is called a simple event. When more than one simple event is combined, this is called a compound event.

In most of our examples, the simple events are usually treated as equally likely. To find the probability of a compound event, add up the probabilities of the simple events that make it up. (all the simple events are mutually exclusive.)

$$P(even) = P(x = 2) + P(x = 4) + P(x = 6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

Or (union)
When the events are mutually exclusive

$$P(A\ or\ B) = P(A) + P(B)$$

When the events are not mutually exclusive: the formula has another term

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

And (intersection)
When A and B are independent events $P(A\ and\ B) = P(A)P(B)$
Independent events are those where if you know the outcome of the first event, the probability of the second event does not change.

Independent event: coin flip and a die roll

Event A = owns power tools
Event B = is a woman

When events are not independent: the formula becomes $P(A\ and\ B) = P(A|B)P(B)$

$P(A|B)$ is read as "the probability of A given B", a conditional probability

If $P(A) = P(A|B)$, then the events are independent

Counting Rules

Factorial is a mathematical operation where all the integers starting at a given number are multiplied together from the given number down to 1.

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

$$0! = 1$$

Count the event space (the number of outcomes in an event), and sample space (the total number of outcomes that could happen), usually in a complex scenario

Multiplication Rule, Permutations, and Combinations

Multiplication Rule: if I have a bunch of outcomes for different events (that are independent), then multiply together the outcomes of the separate events.

You are going on vacation. You are taking 3 pairs of pants, 4 shirts, 2 pairs of shoes, and 2 jackets. How many different outfits can you wear before repeating exactly the same outfit?

$$3 \times 4 \times 2 \times 2 = 48$$

Permutations: can't repeat outcomes, when the order of selection matters
Team sports that have positions for players
Elections, races, contests where the prizes differ

Combinations: can't repeat outcomes, but you don't care about the order
The group has no distinction: committee assignment, raffle prizes for the same value

$$P(n,r) = nPr = P_r^n = P_{n,r}$$

$$C(n,r) = nCr = C_r^n = C_{n,r} = \binom{n}{r}$$

$$P(n,r) = \frac{n!}{(n-r)!}, \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

How many ways can a team of 14 players be put on the field in 9 positions for a baseball game?

$$P(14,9) = \frac{14!}{(14-9)!} = \frac{14!}{5!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 = 726{,}485{,}760$$

A raffle has 4 top prizes worth \$100, \$50, \$25, \$10. If 20 people bought tickets, how many ways can the prizes be given away?

$$P(20,4) = 116{,}280$$

Suppose you have 4 prizes in a raffle that are all the same. If 20 people buy tickets, how many ways can the prizes be given away?

$$\binom{20}{4} = \frac{20!}{4!\,(20-4)!} = 4845$$

You cannot get a decimal from either permutations or combinations.

Probabilities with counting:

Coin flipping multiple coins

How many ways can I get 7 heads on 13 coin flips?

$$\binom{13}{7} = 1716$$

What is the probability of getting 7 heads on 13 coin flips?

Number of ways to flip 13 coins: $2^{13} = 8192$

Probability = $\frac{number\ of\ ways\ to\ get\ 7\ head\ out\ of\ 13\ flips}{total\ ways\ to\ flip\ 13\ coins} = \frac{1716}{8192} \approx 0.20947$

Card problem:
52 cards in the deck, 4 suits (hearts, diamonds, spades, clubs), 13 types of cards (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)
Three-of-a-kind: three of the same type of card: like 3 10's
Full house: two-of-a-kind, one three-of-a-kind

What is the probability of getting a three-of-a-kind of kings in a 5-card poker hand?

$$\frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} = 0.001736$$

What is the probability of getting ANY three-of-a-kind in a 5-card poker hand?

$$13 \times \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} = 0.022569$$

Calculating probabilities from two-way tables:
See Excel