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Math of Finance

Simple Interest

$$I = Prt$$

I is the interest amount, P is the principal (balance in the account), r is the annual interest rate (or rate per period), t is time in years (period).

New amount in account is the Principal + Interest Earned.

$$A = P(1 + rt)$$

Suppose you have $8000 in an account earning simple interest of 4% annually. How much interest have you earned after 2 years? 9 months?

Compounded Interest

Savings account (retirement account) or balloon payment

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

A is the Amount (in account or balance owed), P is the principal, r is the annual interest rate, n is the number of times the interest is calculated per year (compounding periods per year), t is the time in years. (FV is the future value is the amount, and PV is the present value = principal)

Suppose you deposit $400 in a savings account to save for a vacation next year (1 year). The account earns 2% interest compounded monthly. How much is in the account in a year?

Suppose you deposit $10,000 in a CD earning 1.75% interest compounded daily. How much is in the account after 10 years?

Amortization tables use simple interest to calculate the interest or balance during one compounding period at a time. It is best for complex scenarios or short time periods.

Incorporating Payments into loans or savings (sinking funds, annuities). Formulas assume regularity of payments.

Savings

$$FV = \frac{PMT\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$$

Loan

$$PV = \frac{PMT\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]}{\frac{r}{n}}$$

FV = future value is the value in a savings account after time has passed
PMT = payment per compounding period
r is the annual interest rate
n is the number of compounding periods per year
t is time
PV = present value (amount of the loan)

Amortization table for payments

Save $500 per month, in an account paying 5% interest annually, compounded monthly, for 5 years. How much money is in the account at the end? (see Savings sheet)

Suppose you want to buy a car that costs $35,000. And you can afford $500 payments per month. How long will it take to pay off the loan at that pace if the interest charged is 7% compounded monthly? (see Loans sheet)

$500 per month takes 91 months to pay off
$600 per month would take 72 months to pay off… and that is a possible car loan timeframe.

Built-in Formulas (see Excel sheet 5)

Exponential Growth.

Where the growth increases by a multiplier effect each time period

1, 2, 4, 8, 16, 32, 64, 128,…

Multiplier in compound interest is $\left(1+\frac{r}{n}\right)$

To determine if a sequence represents exponential growth (geometric sequence), find the ratio of (all) consecutive pairs (New/Old) to see if the ratio is the same all the time.