Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Let A be the set of letters in the word INCONSEQUENTIAL and let B be set of letters in the word QUINTESSENTIAL. Assume that the universal set is the set of all letters in the English alphabet (of just one case).

    a. List the elements in set A using proper set notation. (5 points)

    b. List the elements in set B using proper set notation. (5 points)

    c. Find $A \cap B$. (5 points)

    d. Find $A \cup B$. (5 points)

    e. What is the number of elements in set A, i.e. $n(A) = |A|$? (3 points)

    f. What are the elements of B^c? (5 points)

2. Answer the following questions about sets:
    a. List the elements in the set $C = \{x|x \text{ is a positive integer less than } 20 \text{ and divisible by } 3\}$.
       (5 points)

b. List the elements in set $D = \{x | x \text{ is an positive integer less than } 1\}$. (5 points)

c. For each of the following questions, answer TRUE or FALSE. (8 points)
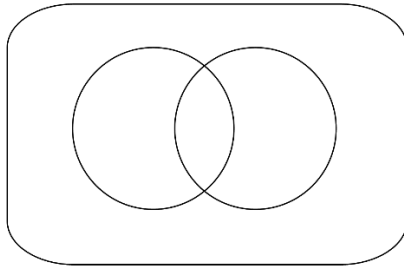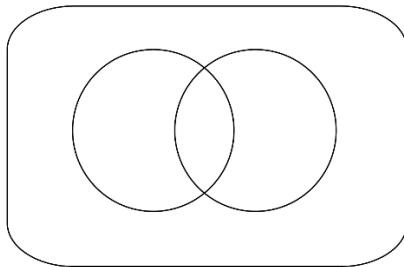   i. $5 \in C$

   ii. $C \cap D = \emptyset$

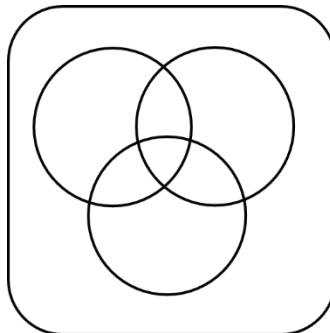   iii. $\emptyset \in C$

   iv. $0 \subset D$

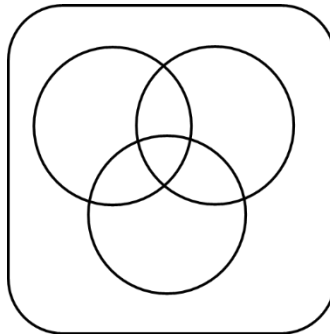3. Draw a Venn Diagram that illustrates each of the following sets.

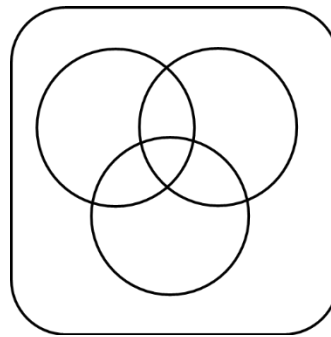a. $A \cup B^C$ (4 points)

b. $B - A$ (4 points)

c. $(A - B^C) \cap C$ (6 points)

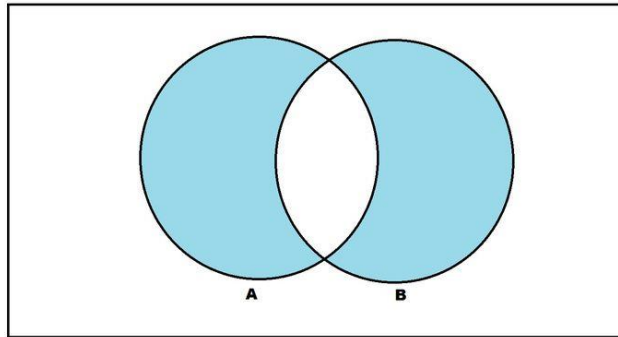d. $(C^c \cup A) \cap B$ (6 points)

e. $(A^C - B) \cup (B - C)$ (8 points)

4. Draw a Venn diagram (of two sets) that displays the indicated relationship between the sets A and B. (5 points each)
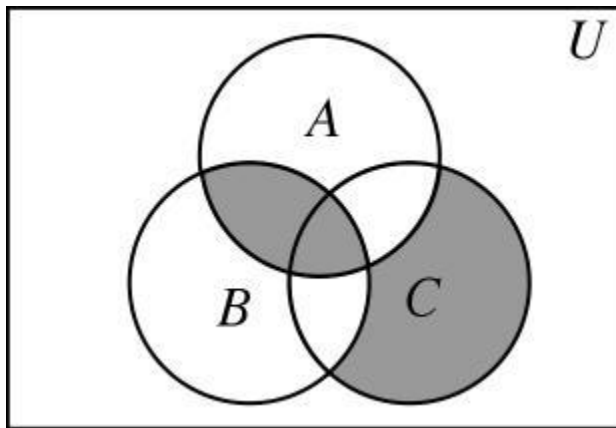   a. A and B are mutually exclusive $(A \cap B = \emptyset)$.

   b. $A \subset B$

5. For each of the following Venn diagrams, write set notation that describes the indicated set. (6 points each)
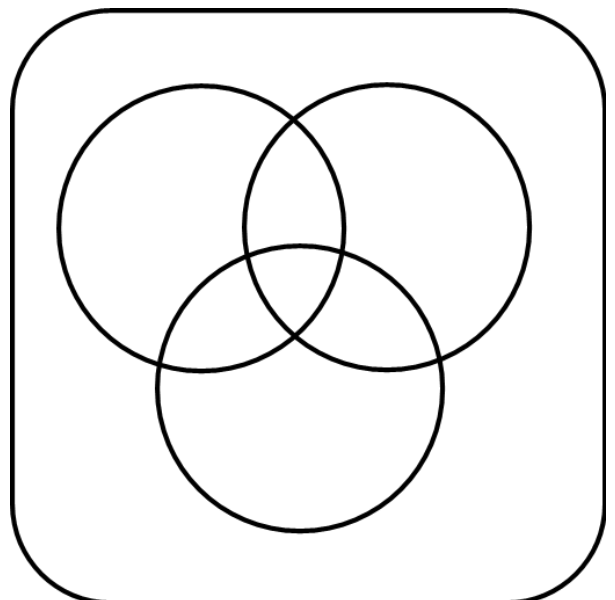

a.

b.

6. In a survey of 250 university students, respondents were asked about which courses they were taking. Let M be the set of all students taking a math course, C be the set of students taking a chemistry course, and P be the set of students taking a physics course. Fill in the Venn diagram below using the following data, and then use the diagram to answer the questions that follow. (20 points)

- 64 had taken mathematics course
- 94 had taken chemistry course
- 58 had taken physics course
- 28 had taken mathematics and physics
- 26 had taken mathematics and chemistry
- 22 had taken chemistry and physics course
- 14 had taken all the three courses

a. Find the number of students who took math only.

b. Find the number of students who took none of these courses.

c. Find the number of students who took exactly two of these courses.

7. Let p be the statement "The cat is sleeping," and let q be the statement "Matilda walks through the woods alone," and r be the statement "Jose bought a new car." Use this information to translate the following symbolic statements into English sentences. (5 points each)

a. $p \vee q$

b. $\sim p \leftrightarrow q$

c. $(p \wedge q) \rightarrow \sim r$

8. Construct truth tables for each of the following statements.

a. $p \rightarrow \sim q$ (12 points)

b. $(p \vee q) \wedge \sim r$ (18 points)

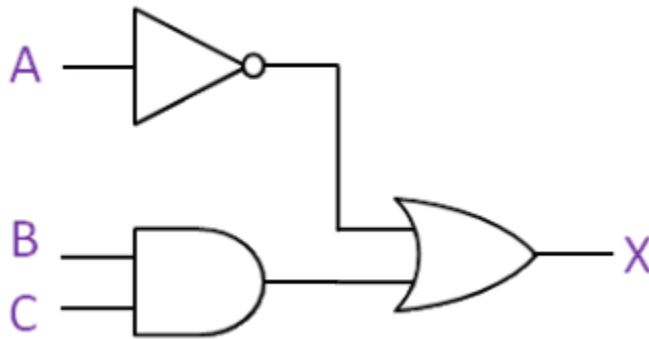|					

9. Explain in your own words the difference between "inclusive or" and "exclusive or". Provide example sentences where inclusive or is meant, and another where exclusive or is meant in English. (8 points)

10. Find the truth value of the logic gates below using the fact that A is True, B is True, and C is False. Determine the final output value. (6 points each)
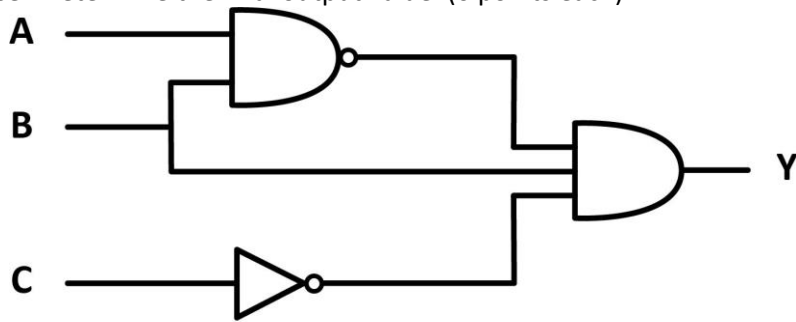
a.

b.

11. Use determine the validity of each of the following arguments. If the argument is invalid, explain why. (7 points each)
   a. All toasters are items made of gold.
      All items made of gold are time-travel devices.
      Therefore, all toasters are time-travel devices.

b. All basketballs are round.
      The Earth is round.
      Therefore, the Earth is a basketball.

12. Translate the following sentences into logical notation. Let p be the statement "The filing cabinet is open," and q be the statement "The car is in the garage." (5 points each)
    a. The filing cabinet is open, and the car is in the garage.

    b. If the car is not in the garage, then the filing cabinet is open.

13. Use a truth table to show that $\sim(p \wedge q)$ is equivalent to $\sim p \vee \sim q$. Clearly indicate which two columns you are matching. (15 points)