

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. For the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 1 & 1 & 1 & 9 \end{bmatrix}$, find an explicit description of Nul A and Col A.

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\text{Col } A = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\begin{aligned} \text{Nul } A: \quad x_1 + 4x_4 = 0 &\rightarrow x_1 = -4x_4 \\ x_2 + 5x_4 = 0 &\rightarrow x_2 = -5x_4 \\ x_3 &= 0 \quad x_3 = 0 \\ &x_4 = x_4 \end{aligned}$$

$$\vec{x} = \begin{bmatrix} -4 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Nul } A = \text{span} \left\{ \begin{bmatrix} -4 \\ -5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

2. Determine if the following sets represent vector spaces.

- a. The set of even functions (i.e. all functions such that $f(x) = f(-x)$).

$$f(x) = f(-x) \quad \& \quad g(x) = g(-x)$$

$$(f+g)(x) = f(x) + g(x) = f(-x) + g(-x) = (f+g)(-x) \quad \text{in set}$$

$$kf(x) = kf(-x) \quad \text{in set}$$

$$0 = 0 \quad \text{in set}$$

is a subspace of all functions

b. $W = \left\{ \begin{bmatrix} a \\ b^2 \end{bmatrix}, a, b \text{ real} \right\}$

is not a subspace

$$(-1) \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix} \quad -4 \neq b^2 \text{ for any real } b.$$