MTH 266, Quiz #7, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. For each of the sets below, determine if the set represents a vector space. If it does, prove it by testing all three conditions for a subspace. If it does not, find at least one case where the vector space conditions are violated.

   a. The set of complex numbers C, in the form *a+bi*, where *a,b* are real numbers.

$X = a+bi,\ c+di = Y \quad a,b,c,d \in \mathbb{R}$

$X+Y = (a+c) + (b+d)i$ in set

$kX = (ka) + (kb)i$ in set

$0 = 0 + 0i$ in set

is a vector space (subspace isomorphic to $\mathbb{R}^2$)

   b. $H = \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}, a+b+c=0\right\}$ $\quad X = \begin{bmatrix} a \\ b \\ c \end{bmatrix},\ Y = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

$X+Y = \begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix} \Rightarrow (a+d)+(b+e)+(c+f) = (a+b+c)+(d+e+f) = 0+0=0$ in set

$kX = \begin{bmatrix} ka \\ kb \\ kc \end{bmatrix} \rightarrow (ka)+(kb)+(kc) = k(a+b+c) = k(0) = 0$ in set

$a=b=c=0 \rightarrow a+b+c=0 \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ in set $\quad$ is a subspace of $\mathbb{R}^3$

   c. $W = \left\{\begin{bmatrix} a & b & 1 \\ a & c & d \end{bmatrix}\right\}$

is not a subspace

for example, if $a=b=c=d \quad \vec{w} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

violates all 3 properties

   d. The set of polynomials of less than or equal to degree 4 of the form $p(t) = a_1t + a_2t^2 + a_4t^4$ as a subspace of $\mathbb{P}_4$. $\quad q(t) = b_1t + b_2t^2 + b_4t^4$

$p(t) + q(t) = (a_1+b_1)t + (a_2+b_2)t^2 + (a_4+b_4)t^4$ in set

$kp(t) = (ka_1)t + (ka_2)t^2 + (ka_4)t^4$ in set

$p(t) = 0$ in set if $a_1=0,\ a_2=0,\ a_4=0$