MTH 266, Quiz #6, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Calculate the determinant below using row operations.

$$\begin{vmatrix} 1 & 3 & 5 & 2 \\ 0 & 1 & -4 & -1 \\ 3 & -2 & 4 & 7 \\ 2 & 5 & 1 & 3 \end{vmatrix}$$

$-3R_1 + R_3 \to R_3$, $-2R_1 + R_4 \to R_4$

$$\begin{bmatrix} 1 & 3 & 5 & 2 \\ 0 & 1 & -4 & -1 \\ 0 & -11 & -11 & 1 \\ 0 & -1 & -9 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & -1 \\ -11 & -11 & 1 \\ -1 & -9 & -1 \end{bmatrix}$$

$R_1 + R_3 \to R_3$, $11R_1 + R_2 \to R_2$

$$\begin{bmatrix} 1 & -4 & -1 \\ 0 & -55 & -10 \\ 0 & -13 & -2 \end{bmatrix} \to \begin{vmatrix} -55 & -10 \\ -13 & -2 \end{vmatrix} = 110 - 130 = -20$$

2. If $\det(A) = 5, \det(B) = -2, \det(C) = 0$, use properties of determinant to find the following (if possible). Each matrix is 4×4.
   a. $\det(AB)$ -10
   b. $\det(A^2C)$ 0
   c. $\det(A^{-1})$ $\frac{1}{5}$
   d. $\det(-CB)$ $(-1)^4(0)(-2) = 0$
   e. $\det\left(\frac{1}{2}C^T\right)$ 0

3. Use Cramer's rule to solve the system $\begin{cases} x_1 + 3x_2 = 5 \\ x_1 + x_2 = -4 \end{cases}$

$$|A| = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 1 - 3 = -2$$

$$|A_1| = \begin{vmatrix} 5 & 3 \\ -4 & 1 \end{vmatrix} = 5 + 12 = 17$$

$$|A_2| = \begin{vmatrix} 1 & 5 \\ 1 & -4 \end{vmatrix} = -4 - 5 = -9$$

$x_1 = \frac{-17}{2}$

$x_2 = \frac{9}{2}$