

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Find the critical point(s) of the graph $f(x, y) = x^2 + 2xy + 2y^2 - 6y + 2$ and characterize the point as a maximum, minimum, saddle point, or cannot be determined.

$$\begin{aligned} f_x &= 2x + 2y \\ f_{xx} &= 2 \\ f_{xy} &= 2 \\ f_y &= 2x + 4y - 6 \\ f_{yy} &= 4 \end{aligned}$$

$$\begin{aligned} 2x + 2y &= 0 \rightarrow x = -y \\ 2x + 4y - 6 &= 0 \rightarrow 2x - 4x - 6 = 0 \\ -2x &= 6 \rightarrow x = -3 \\ y &= 3 \\ &\text{critical pt.} \end{aligned}$$

$$\begin{aligned} D &= f_{xx} f_{yy} - (f_{xy})^2 = \\ (2)(4) - (2)^2 &= 8 - 4 = 4 \text{ max/min} \end{aligned}$$

$$f_{xx} > 0 \cup (-3, 3) \text{ is a minimum}$$

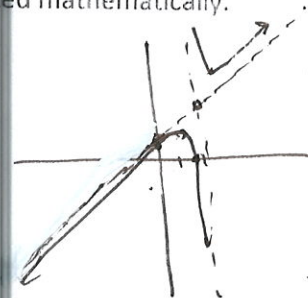
2. Analyze and sketch the graph of the function $f(x) = \frac{x^2 - 3}{2x - 4}$. You must apply appropriate calculus techniques to support your sketch. While you may verify your analysis in your calculator, credit will only be awarded for the portions that are properly supported mathematically.

$$\begin{aligned} \text{VA: } 2(x-2) &= 0 \quad x=2 \\ \text{SA: } \frac{1}{2}x+1 &+ \frac{1}{2x-4} \end{aligned}$$

$$\begin{array}{r} 2x-4 \overline{) x^2+0x-3} \\ \underline{-x^2+2x} \\ 2x-3 \\ \underline{-2x+4} \\ 1 \end{array}$$

X-ints at $\pm\sqrt{3}$

Y-int $\frac{3}{4}$



$$f''(x) = \frac{2x(2x-4) - 2(x^2-3)}{(2x-4)^2} = \frac{4x^2 - 8x - 2x^2 + 6}{(2x-4)^2} = \frac{2x^2 - 8x + 6}{(2x-4)^2} = \frac{2(x^2 - 4x + 3)}{(2x-4)^2} = \frac{2(x-3)(x-1)}{(2x-4)^2}$$

