MTH Homework #3 Key

1a. $f(x,y) = x^2 - 3xy$

$f_x = 2x - 3y \qquad f_y = -3x$

b. $f(x,y) = \ln x + y^3$

$f_x = \frac{1}{x} \qquad f_y = 3y^2$

c. $f(x,y) = xe^{\sqrt{y}}$

$f_x = e^{\sqrt{y}} \qquad f_y = \frac{x}{2\sqrt{y}} e^{\sqrt{y}}$

d. $f(x,y) = x^3 - 3xy^2$

$f_x = 3x^2 - 3y^2 \qquad f_y = -6xy$

e. $f(x,y) = 3^x + 7xy$

$f_x = 3^x \ln 3 + 7y \qquad f_y = 7x$

f. $f(x,y) = x^2 + 4x + y^2$

$f_x = 2x + 4 \qquad f_y = 2y$

g. $f(x,y) = \frac{1}{y+x^2} = (y+x^2)^{-1}$

$f_x = -1(y+x^2)^{-2}(2x) \qquad f_y = -1(y+x^2)^{-2}(1)$

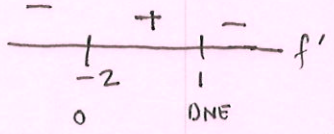
2. $f(x) = 9x - 3x^2 - x^3 = -x(x^2 + 3x - 9) \quad x = \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm \sqrt{45}}{2} = \frac{-3 \pm 3\sqrt{5}}{2}, 0$ (x-intercepts)

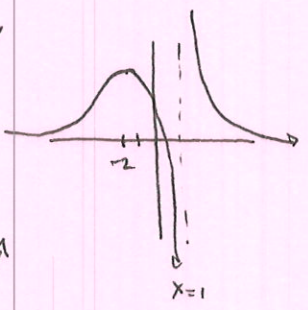
$f'(x) = 9 - 6x - 3x^2 = 3(3 - 2x - x^2) = -3(x^2 + 2x - 3)$

$-3(x+3)(x-1)$

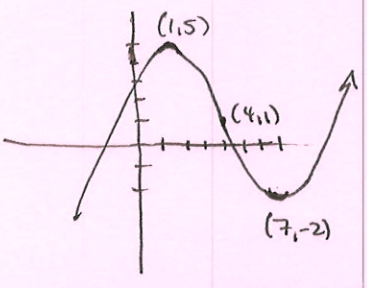
Critical points $x = -3, 1$

3a.

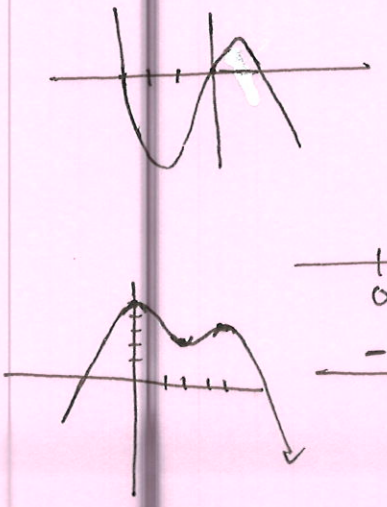


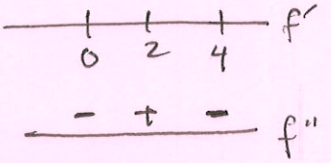


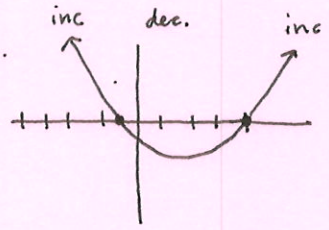
b.

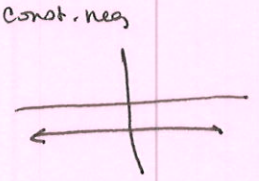


c.





4. a. 

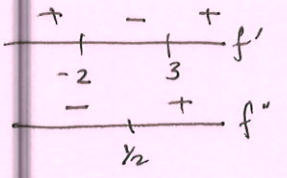
b. 

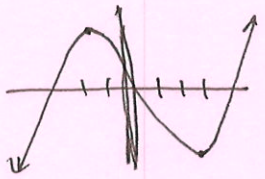
5. $f(x) = 2x^3 - 3x^2 - 36x + 28$

a. $f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2)$

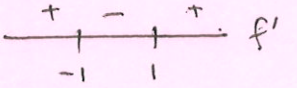
$x = 3, -2$

$f''(x) = 12x - 6 = 6(2x-1) \quad x = +1/2$





b. $f(x) = \dfrac{8x}{x^2+1} \qquad f'(x) = \dfrac{8(x^2+1) - 2x(8x)}{(x^2+1)^2} = \dfrac{8 - 8x^2 = 8(1-x^2)}{(x^2+1)^2}$

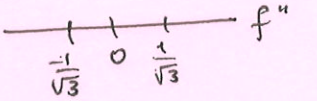


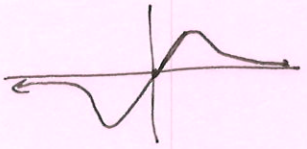
$f''(x) = \dfrac{8(-2x)(x^2+1)^2 - 2(x^2+1)(2x)8(1-x^2)}{(x^2+1)^4}$

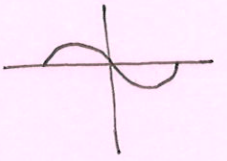
$= \dfrac{8(x^2+1)[-2x(x^2+1) - 4x(1-x^2)]}{(x^2+1)^{4/3}} = \dfrac{-16x[x^2+1 - 2(1-x^2)]}{(x^2+1)^3} =$

$\dfrac{-16x[x^2+1-2+2x^2]}{(x^2+1)^3} = \dfrac{-16x[3x^2-1]}{(x^2+1)^3}$

$x = 0, \; x = \pm\frac{1}{\sqrt{3}}$







c. $f(x) = -x\sqrt{1-x^2} \qquad f'(x) = -\sqrt{1-x^2} - x\,\frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$\dfrac{x^2 - (\sqrt{1-x^2})^2}{\sqrt{1-x^2}} = \dfrac{x^2 - (1-x^2)}{\sqrt{1-x^2}} = \dfrac{2x^2-1}{\sqrt{1-x^2}}$

$x = \pm\frac{1}{\sqrt{2}}$ extrema critical pts

$x = \pm 1$ cusp? undef.

$f''(x) = \dfrac{4x\sqrt{1-x^2} - \frac{1}{2}(1-x^2)^{-1/2}(2x^2-1)(-2x)\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}}{(1-x^2)}$

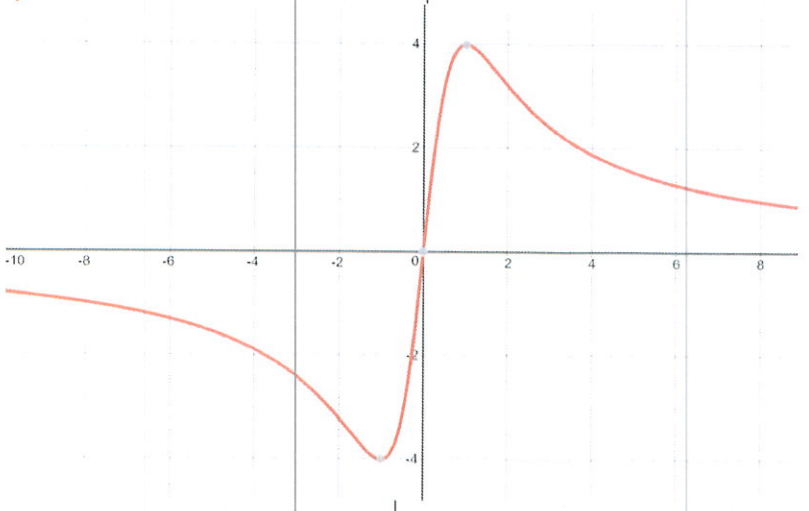
$= \dfrac{4x(1-x^2) + 2x^3 - x}{(1-x^2)^{3/2}} = \dfrac{4x - 4x^3 + 2x^3 - x}{(1-x^2)^{3/2}} = \dfrac{3x - 2x^3}{(1-x^2)^{3/2}} = \dfrac{x(3-2x^2)}{(1-x^2)^{3/2}}$

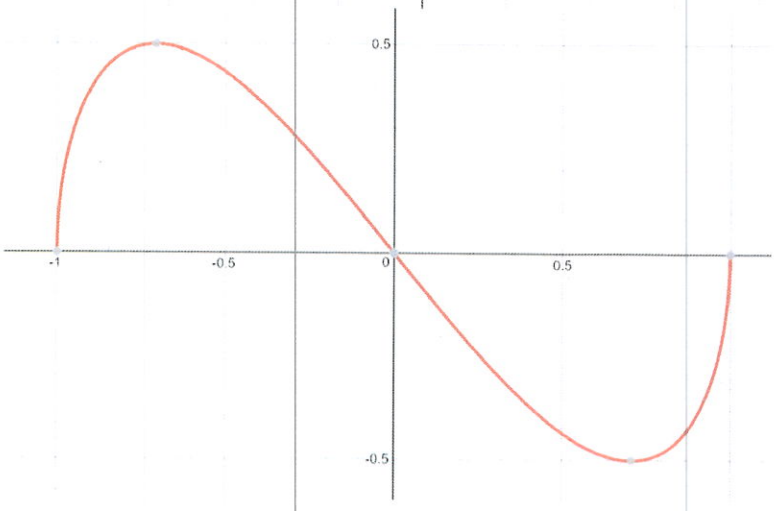
$x = 0$

$x = \pm\sqrt{\frac{3}{2}}$

$x = \pm 1$ undef.


50
-50
-5
-4
-3
-2
-1
0
1
2
3
4
5


4
2
-2
-4
-10
-8
-6
-4
-2
0
2
4
6
8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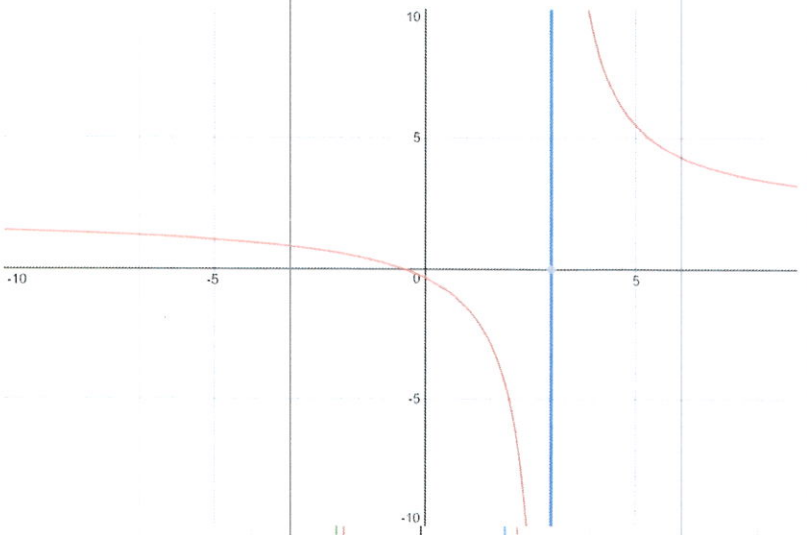
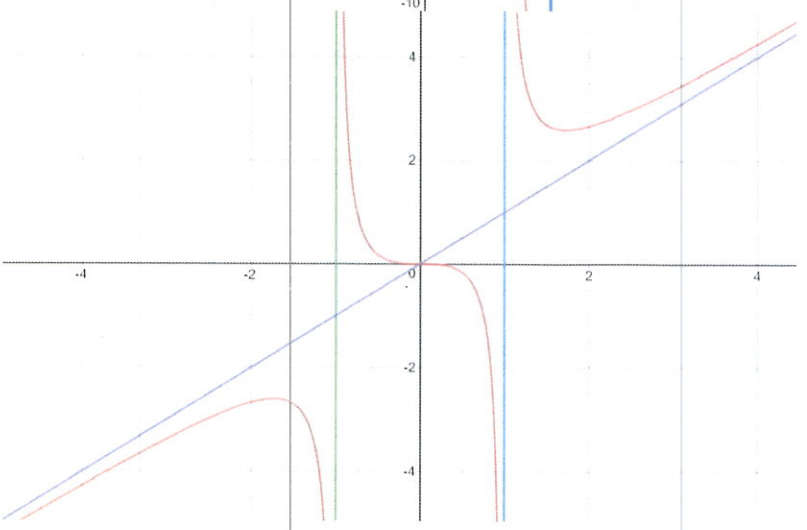
-0.5
-1
-0.5
0
0.5


10
5
-10
-5
0
5
-5
-10

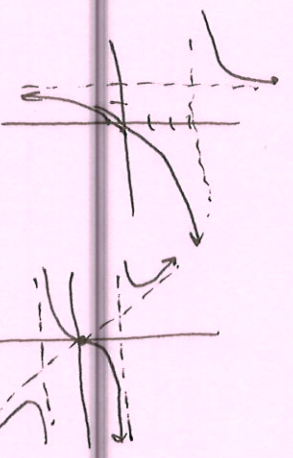

4
2
-4
-2
0
2
4
-2
-4

6a. $f(x) = \frac{2x+1}{x-3}$

HA: $y = 2$

VA: $x = 3$

intercepts $(0, -1/3)$ $(-1/2, 0)$



b. $g(x) = \frac{x^3}{x^2-1}$

SA: $x = y$

VA: $x = 1, x = -1$

intercept $(0,0)$

7 a. $f(x,y) = x^2 + xy + 3y^2 + 11x$

$f_x = 2x + y + 11 = 0 \Rightarrow y = -2x - 11$

$f_y = x + 6y = 0 \quad x = -6y$

$y = -2(-6y) - 11$

$y = 12y - 11 \rightarrow -11y = -11$

$y = 1, \; x = -6$

$f_{xx} = 2$

$f_{xy} = 1$

$f_{yy} = 6$

D: $(2)(6) - 1^2 = 11$

min or max

$f_{xx} > 0 \cup$ Concave up $\Rightarrow$ min @ $(-6, 1)$

b. $f(x,y) = x^3 + y^3 - 6xy$

$f_x = 3x^2 - 6y = 0 \quad 3x^2 = 6y \Rightarrow x^2 = 2y \Rightarrow y = \frac{1}{2}x^2$

$f_y = 3y^2 - 6x = 0 \quad 3y^2 = 6x \Rightarrow y^2 = 2x \quad (\frac{1}{2}x^2)^2 = 2x \rightarrow \frac{1}{4}x^4 = 2x$

$x^4 = 8x$

$x^4 - 8x = 0$

$x(x^3 - 8) = 0$

$x = 0, \; x = 2$

$y = 0 \quad y = 2$

$f_{xx} = 6x$

$f_{yy} = 6y$

$f_{xy} = -6$

$D(0,0) = 0(0) - (-6)^2 = -36$

saddle point @ $(0,0)$

$D(2,2) = 12(12) - (-6)^2 = 144 - 36 > 0$

min/max

$f_{xx} > 0 \cup$ concave up minimum @ $(2,2)$

c. $f(x,y) = 4xy - x^3 - 2y^2$

$f_x = 4y - 3x^2 = 0 \rightarrow 4y = 3x^2 \rightarrow y = \frac{3}{4}x^2 \quad \frac{3}{4}x^2 = x \rightarrow 3x^2 - 4x = 0$

$f_y = 4x - 4y = 0 \quad x = y$

$x(3x - 4) = 0$

$x = 0, \; x = 4/3$

$y = 0 \quad y = 4/3$

$f_{xx} = -6x$

$f_{yy} = -4$

$f_{xy} = 4$

$D(0,0) = 0(-4) - (4)^2 = -16$

Saddle point @ $(0,0)$

$D(4/3, 4/3) = (-8)(-4) - 4^2 = 32 - 16 = 16 > 0$

min/max

$f_{xx} < 0 \cap$ Concave down maximum @ $(4/3, 4/3)$

8. $P(a,n) = -5a^2 - 3n^2 + 48a - 4n + 2an + 290$

$P_a = -10a + 48 + 2n = 0 \Rightarrow n = 5a - 24 \Rightarrow n = 5(5) - 24 = 1 \quad (5,1)$

$P_n = -6n - 4 + 2a = 0$

$-6(5a-24) - 4 + 2a = 0 \rightarrow -30a + 144 - 4 + 2a = 0 \rightarrow -28a = -140$

$a = 5$

$P_{aa} = -10$

$P_{nn} = -6$

$P_{an} = 2$

$D = (-10)(-6) - 2^2 = 60 - 4 = 56 > 0$

min/max

$P_{aa} < 0 \cap$ concave down maximum @ $(5,1)$