MTH 261, Exam #2, Part I, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may **not** use a calculator for this section of the exam and all answers without work will receive minimal credit. Use exact answers.

1. Find the absolute extrema of the function $f(x) = x^4 - 2x^2 + 5$ on the interval $[-2,2]$. (10 points)

$f'(x) = 4x^3 - 4x = 4x(x^2-1)$

$x = 0, 1, -1$

$f(0) = 5$

$f(1) = 4$

$f(-1) = 4$ } absolute minimum

$f(-2) = 13$

$f(2) = 13$ } absolute maximum

2. Find the equation of the tangent line to the implicitly defined function $x^4 - x^2y^3 = 12$ at the point $(-2,1)$. (12 points)

$4x^3 - 2xy^3 - 3x^2y^2y' = 0$

$4x^3 - 2xy^3 = 3x^2y^2y'$

$\frac{4x^3 - 2xy^3}{3x^2y^2} = y' \Rightarrow 5/3 @ (-2,1)$

$y - 1 = \frac{5}{3}(x+2)$

$y - 1 = \frac{5}{3}x + \frac{10}{3}$

$+1 \qquad +1$

$y = \frac{5}{3}x + \frac{13}{3}$

3. Integrate $\int \frac{4}{\sqrt[5]{x}} + \frac{3}{4}e^{6x} - \frac{7}{x}dx$. (8 points)

$$\int 4x^{-1/5} + \frac{3}{4}e^{6x} - \frac{7}{x}\,dx =$$

$$4 \cdot \frac{5}{4} \cdot x^{4/5} + \frac{3}{4} \cdot \frac{1}{6}e^{6x} - 7\ln x + C$$

$$5x^{4/5} + \frac{1}{8}e^{6x} - 7\ln x + C$$

$$5\sqrt[5]{x^4} + \frac{1}{8}e^{6x} - 7\ln x + C$$

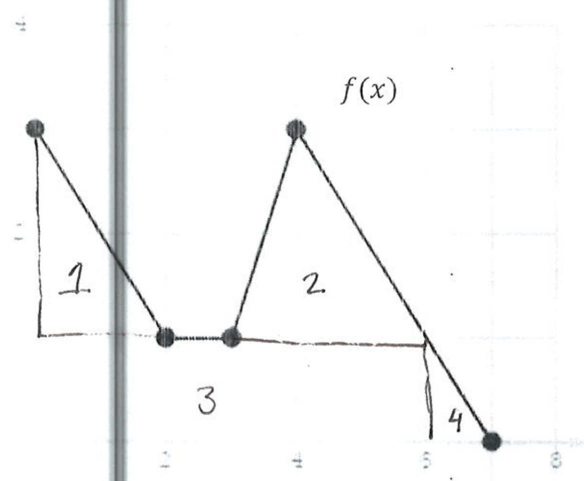
4. Find $f(x)$ if $f'(x) = 8x^2 + 4x - 2$. Find the constant of integration if $f(0) = 6$. (10 points)

$$\int 8x^2 + 4x - 2\,dx = \frac{8}{3}x^3 + 2x^2 - 2x + C$$

$$\frac{8}{3}(0)^3 + 2(0)^2 - 2(0) + C = 6 \Rightarrow C = 6$$

$$f(x) = \frac{8}{3}x^3 + 2x^2 - 2x + 6$$

5. Use graph to the right to evaluate the integral $\int_0^7 f(x)dx$ geometrically. (12 points)



$A_1 = \frac{1}{2}(2)(2) = 2$

$A_2 \ \frac{1}{2}(2)(3) = 3$

$A_3 = 5 * 1 = 5$

$A_4 = \frac{1}{2}(1)(1) = \frac{1}{2}$

$10\frac{1}{2} = \frac{21}{2}$

6. Find the error(s) in the following work. Identify each error, explain why it is incorrect, and make an appropriate correction. [Hint: there is at least one mistake.] (10 points)

$$\int_1^2 lnx - e^x dx = \left[\frac{1}{x} - e^x\right]_1^2$$
$$= \left(\frac{1}{2} - e^2\right) - (1 - e^1)$$
$$= e - e^2 - \frac{1}{2}$$

$\int \ln x\, dx$ $u = \ln x$ $dv = dx$

$du = \frac{1}{x} dx$ $v = x$

$x \ln x - \int 1\, dx =$

$x \ln x - x$

$\int \ln x\, dx \neq \frac{1}{x}$

this is the derivative, not the antiderivative

$\int_1^2 \ln x - e^x dx = [x \ln x - x - e^x |_1^2$

$= 2\ln 2 - 2 - e^2 - \cancel{1 \ln 1} + 1 + e^1 =$

$2\ln 2 - 1 + e - e^2$

7. Find the area under the curve $f(x) = x^3 - 3x$ over the interval $[-1,1]$. (10 points)

$\int_{-1}^1 |x^3 - 3x| dx = -\int_0^1 x^3 - 3x\, dx + -\int_0^{-1} x^3 - 3x\, dx$

$5/4 + 5/4 = \frac{10}{4} = \frac{5}{2}$

w/o splitting, integral is zero

8. Find the area bounded by $f(x) = x$ and $g(x) = \sqrt[4]{x}$. Sketch the graph. (10 points)

$\int_0^1 x^{1/4} - x\, dx =$

$\frac{4}{5} x^{5/4} - \frac{1}{2} x^2 |_0^1 =$

$\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

9. Integrate. (10 points each)

a. $\int x^3 e^x dx$ $= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

+/-	u	dv
+	x^3	e^x
-	$3x^2$	e^x
+	$6x$	e^x
-	6	e^x
+	0	e^x

b. $\int \ln x \, dx$ (See #6)

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$$x \ln x - \int \frac{1}{x} \cancel{x} \, dx = x \ln x - \int 1 \, dx = x \ln x - x + C$$

c. $\int \frac{e^{1/x}}{x^2} dx$

$u = 1/x$

$du = -\frac{1}{x^2} dx$

$\int -e^u du = -e^{1/x} + C$

d. $\int \frac{1}{1+7x} dx$

$u = 1 + 7x$

$du = 7 dx$

$\frac{1}{7} du = dx$

$\frac{1}{7} \int \frac{1}{u} du = \frac{1}{7} \ln |1+7x| + C$

MTH 261, Exam #2, Part II, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may use a calculator for this section of the exam and all answers without work will receive minimal credit. Use exact answers unless the problem begins with decimals or you are specifically asked to round.

10. Riverside Appliances is marketing a new refrigerator. It determines that in order to sell x refrigerators, the price per suit must be $p = 280 - 0.4x$. It also determines that the total cost of producing x refrigerators is given by $C(x) = 5000 + 0.6x^2$.

a. Find the total revenue $R(x)$. (5 points)

$R(x) = 280x - 0.4x^2$ $R(x) = xp$

b. Find the total profit $P(x)$. (5 points)

$P(x) = R - C = 280x - 0.4x^2 - (5000 + 0.6x^2) =$

$280x - x^2 - 5000 = P(x)$

c. How many refrigerators must the company produce and sell in order to maximize profit? (6 points)

140

$P'(x) = 280 - 2x$

$= 0$

$280 = 2x$

$x = 140$

d. What is the maximum profit? (5 points)

$P(140) = \$14,600$

e. What is the price per refrigerator that must be charged in order to maximize profit? (6 points)

$p = 280 - 0.4(140) = \$224$

11. Suppose that the price p in dollars and number of sales x of a certain item follow the equation $5p + 4x + 2px = 60$. Suppose also that p and x are both functions of time, measured in days. Find the rate at which x is changing $\left(\frac{dx}{dt}\right)$ when $x = 3, p = 5, \frac{dp}{dt} = 1.5$. (12 points)

$$5\frac{dp}{dt} + 4\frac{dx}{dt} + 2\frac{dp}{dt}x + 2p\frac{dx}{dt} = 0$$

$$5(1.5) + 4\frac{dx}{dt} + 2(1.5)(3) + 2(5)\frac{dx}{dt} = 0$$

$$7.5 + 14\frac{dx}{dt} + 9 = 0$$

$$16.5 = -14\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{-33}{28} \approx -1.18$$

12. Suppose that P_0 is invested in the Mandelbrot Bond Fund for which interest is compounded continuously at 5.9% per year. That is the balance P grows at the rate given by $\frac{dP}{dt} = 0.059P$.

a. Find the function that satisfies the equation in terms of P_0 and 0.059. (5 points)

$$P(t) = P_0 e^{0.059t}$$

b. Suppose that \$1000 is invested. What is the balance in the account after 1 year? (5 points)

$$P(t) = 1000e^{0.059t} \Rightarrow P(1) = 1060.78$$

c. What is the balance in the account after 2 years? (6 points)

$$P(2) = 1125.24$$

d. When will an investment of \$1000 double itself? (6 points)

$$t = \frac{\ln 2}{0.059} = 11.75 \text{ years}$$

13. Carbon-14 has a decay rate that is modeled by the equation $\frac{dN}{dt} = -0.00012097N$, where t is in years. How old is an ivory tusk if 40% of its original Carbon-14 remains? (12 points)

$$.40 = e^{-.00012097t}$$

$$\frac{\ln .40}{-.00012097} = t = 7574.53 \text{ years}$$

14. The elasticity of demand is given by $E(x) = -\frac{xD'(x)}{D(x)}$. Find the elasticity for $D(x) = \sqrt{600 - x}$, at $x = 100$. (10 points)

$$D'(x) = \frac{1}{2}(600-x)^{-1/2}(-1) = \frac{-1}{2\sqrt{600-x}}$$

$$E(x) = -\frac{x\left(-\frac{1}{2\sqrt{600-x}}\right)}{\sqrt{600-x}} = \frac{x}{2(600-x)}$$

$$E(100) = \frac{100}{2(500)} = \frac{1}{10} = 0.1$$

15. Approximate the area under the curve $f(x) = \frac{1}{x^2}$ on the interval $[1,7]$ by computing the area under 6 rectangles (using the left-hand rule). (15 points)

$\Delta x = \frac{7-1}{6} = 1$ $\qquad x_0 = 1,\ x_1 = 2,\ x_3 = 3,\ \ldots\ x_6 = 7,$

$$\sum_{i=0}^{5} f(x_i)\Delta x = \left[\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36}\right](1) =$$

$$\doteq 1.49$$