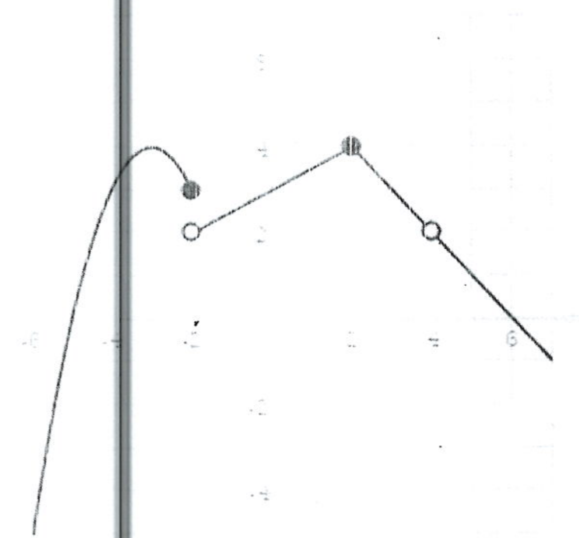
MTH 261, Exam #1, Part I, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may **not** use a calculator for this section of the exam and all answers without work will receive minimal credit. Use exact answers.

1. Use the graph below to answer the following questions. (3 points each)



a. $\lim_{x\to-2^+} F(x)$ 2

b. $\lim_{x\to-2^-} F(x)$ 3

c. $\lim_{x\to2^+} F(x)$ 4

d. $\lim_{x\to2^-} F(x)$ 4

e. $\lim_{x\to2} F(x)$ 4

f. $\lim_{x\to4} F(x)$ 2

g. $F(-2)$ 3

h. $F(2)$ 4

2. Use the definition of the derivative $f'(x) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ to find the derivative of the following functions. (8 points each)

a. $f(x) = -2x + 3$

$$\lim_{h\to0}\frac{-2(x+h)+3-(-2x+3)}{h} = \lim_{h\to0}\frac{\cancel{-2x}-2h\cancel{+3}\cancel{+2x}\cancel{-3}}{h} = \lim_{h\to0}\frac{-2\cancel{h}}{\cancel{h}} = -2$$

b. $f(x) = x^2 + 5x - 11$

$$\lim_{h \to 0} \frac{(x+h)^2 + 5(x+h) - 11 - (x^2+5x-11)}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h - \cancel{11} - \cancel{x^2} - \cancel{5x} + \cancel{11}}{h} = \lim_{h \to 0} \frac{2xh + h^2 + 5h}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x+h+5)}{\cancel{h}} = \lim_{h \to 0} 2x+h+5 = 2x+5$$

3. Find the equation of the tangent line at $x = 1$ of the graph $f(x) = \frac{4}{x^2}$. (12 points) $(1,4)$

$4x^{-2}$ $f(1)$

$$f'(x) = -8x^{-3} = \frac{-8}{x^3}$$

$$f'(1) = -8$$

$$y - 4 = -8(x-1)$$

$$y - 4 = -8x + 8 \quad \to \quad y = -8x + 12$$
$$+4 \qquad\quad +4$$

4. Find the derivative of $f(x) = \sqrt[3]{2x + (x^2 + x)^4}$. (10 points)

$$f'(x) = \tfrac{1}{3}(2x + (x^2+x)^4)^{-2/3}[2 + 4(x^2+x)^3(2x+1)]$$

$$= \frac{2 + 4(x^2+x)^3(2x+1)}{3\sqrt[3]{[2x+(x^2+x)^4]^2}}$$

5. Find f_{xy} and f_{yy} for the function $f(x, y) = x^4y^3 - x^2y^3$. (10 points)

$f_x = 4x^3y^3 - 2xy^3$

$f_{xy} = 12x^3y^2 - 6xy^2$

$f_y = 3x^4y^2 - 3x^2y^2$

$f_{yy} = 6x^4y - 6x^2y$

6. Use the first derivative test (create a sign chart) to determine whether each of the critical points of the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 4x - 1$ is a maximum or a minimum (or neither). (12 points)

$f'(x) = x^2 - 4x + 4 = (x-2)^2 = 0 \quad x = 2$

$f''(x) = 2x - 4 = 2(x-2) = 0 \quad x = 2$

f': $+$ | 2 | $+$

f'': $-$ | 2 | $+$

$x = 2$

neither

7. Use the second derivative test to determine whether each of the critical points of the graph $g(x) = x^4 - 4x^3 + 10$ is a maximum or a minimum (or neither). (10 points)

$g'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0 \quad x = 0, x = 3$

$g''(x) = 12x^2 - 24x = 12x(x-2) = 0 \quad x = 0, x = 2$

f': $-$ | 0 | $-$ | 3 | $+$

f'': $+$ | 0 | $-$ | 2 | $+$

$x = 0$ neither

$x = 3$ min (concave up at 3 ∪)

8. Sketch the graph of the function $f(x) = \frac{x^2+1}{x}$ using calculus and algebraic techniques. Note any intercepts, critical points, and asymptotes. (16 points)

VA: $X=0$

SA: $X \overline{\smash{)}X^2+0X+1}$ with quotient X; $-X^2$; remainder 1

$y = X$: SA , $y = X + \frac{1}{X}$

no y-intercept

no x-intercept

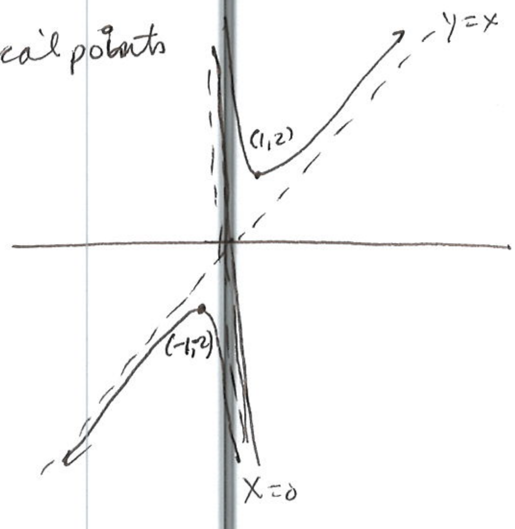
$f'(x) = 1 - \frac{1}{X^2} = 0$

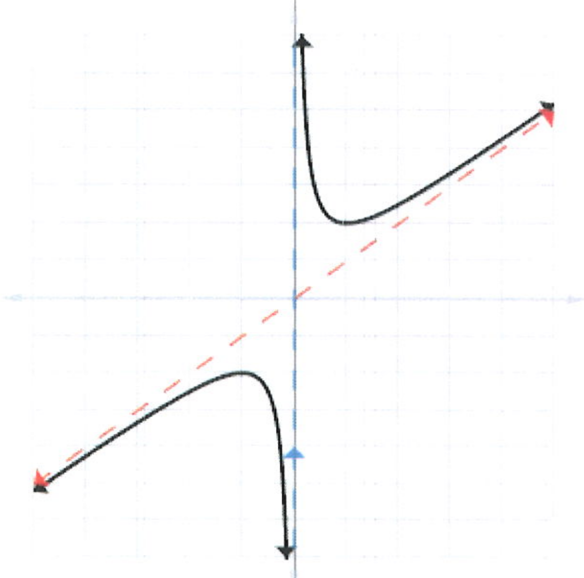
$1 = \frac{1}{X^2} \Rightarrow X^2 = 1$

$X = \pm 1$ critical points

$f(1) = \frac{2}{1} = 2$

$f(-1) = \frac{2}{-1} = -2$





MTH 261, Exam #1, Part II, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may use a calculator for this section of the exam and all answers without work will receive minimal credit. Use exact answers unless the problem begins with decimals or you are specifically asked to round.

9. Evaluate $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$ numerically. Show a table of values to support your evaluation of the limit. (8 points)

x	26	25.1	25.01	25.001	24.999	24.99	24.9	24
$f(x)$	0.9902	0.999	.09999	0.1	0.1	0.10001	.1001	0.10102

$$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} = 0.1 \text{ or } \frac{1}{10}$$

10. In t seconds, an object dropped from a certain height will fall $s(t)$ feet, where $s(t) = 16t^2$.

a. Find $s(5), s(3)$. (4 points)

$$s(5) = 16(5)^2 = 16 \times 25 = 400$$

$$s(3) = 16(3)^2 = 16 \times 9 = 144$$

b. Find the average rate of change (or average speed) over this period from 3 seconds to 5 seconds. (6 points)

$$\frac{400-144}{5-3} = \frac{256}{2} = 128 \text{ ft/sec}$$

11. Find the derivative of the functions below. (8 points each)

a. $f(x) = \frac{4}{7x^3} - \sqrt[3]{x^5}$

$$\frac{4}{7}x^{-3} - x^{5/3}$$

$$f'(x) = -\frac{12}{7}x^{-4} - \frac{5}{3}x^{2/3}$$

$$= \frac{-12}{7x^4} - \frac{5\sqrt[3]{x^2}}{3}$$

b. $g(x) = e^{3x} - \frac{4}{5}e^{x^3} + 5^x$

$$g'(x) = 3e^{3x} - \frac{12}{5}x^2e^{x^3} + (\ln 5)5^x$$

c. $h(x) = [\ln(e^x - x)]^3$

$$h'(x) = 3[\ln(e^x - x)]^2 \cdot \frac{e^x - 1}{e^x - x}$$

d. $F(x) = \frac{8x+\sqrt{x}}{x^3-4x}$

$$F'(x) = \frac{\left(8 + \frac{1}{2\sqrt{x}}\right)(x^3 - 4x) - (3x^2 - 4)(8x + \sqrt{x})}{(x^3 - 4x)^2}$$

e. $G(x) = x^3\sqrt{5x+2}$

$$G'(x) = 3x^2\sqrt{5x+2} + \frac{5x^3}{2\sqrt{5x+2}}$$

12. Find the first 4 derivatives of $g(x) = x^4 - 3x^3 - 7x^2 - 6x + 9$. (8 points)

$g'(x) = 4x^3 - 9x^2 - 14x - 6$

$g''(x) = 12x^2 - 18x - 14$

$g'''(x) = 24x - 18$

$g^{iv}(x) = g^{(4)}(x) = 24$

13. Find the first partial derivatives of each of the functions. (8 points each)

a. $f(x,y) = 7xy^2 + 5xy - 2y$

$f_x = 7y^2 + 5y$

$f_y = 14xy + 5x - 2$

b. $g(x,y,z) = e^{xy} - z\ln y + x^{1.2}z^{0.8}$

$g_x = ye^{xy} + 1.2x^{0.2}z^{0.8}$

$g_y = xe^{xy} - \frac{z}{y}$

$g_z = -\ln y + 0.8x^{1.2}z^{-0.2}$

14. Find the critical point(s) and determine whether each is a maximum, minimum, saddle point, or cannot be determined, for the function $f(x,y) = x^2 + xy + y^2 - 5y$. (10 points)

$f_x = 2x + y = 0 \quad y = -2x$

$f_y = x + 2y - 5 = 0$

$x + 2(-2x) = 5$

$x - 4x = -3x = 5$

$x = \frac{-5}{3} \quad y = \frac{10}{3}$

$\left(-\frac{5}{3}, \frac{10}{3}\right)$

min

$f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = 1$

$D: (2)(2) - 1^2 = 3$

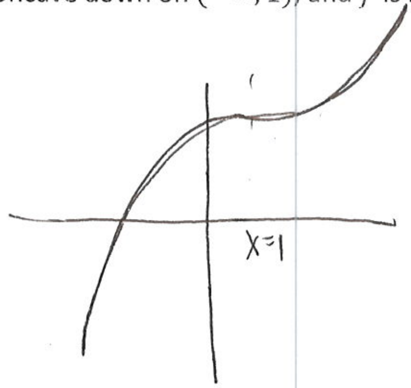
max or min

$f_{xx} > 0$ ∪ = min

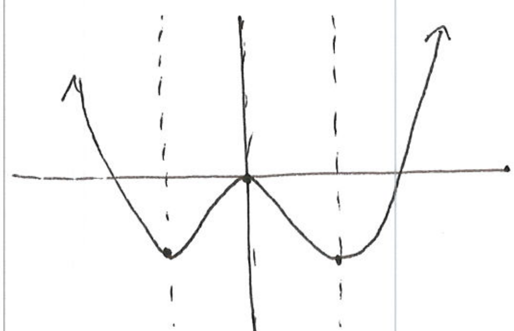
concave up

15. Sketch a graph of a function with the following properties: (6 points each)

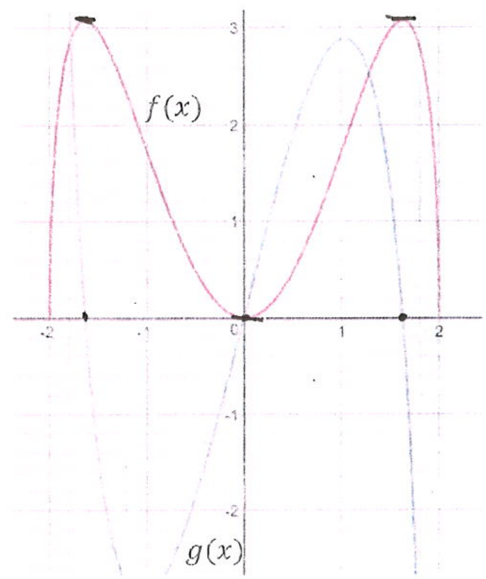
a. f is increasing and concave down on $(-\infty, 1)$, and f is increasing and concave up on $(1, \infty)$.



b. $f'(-1) = 0, f''(-1) > 0, f(-1) = -2; f'(1) = 0, f''(1) > 0, f(1) = -2;\ f'(0) = 0, f''(0) < 0, f(0) = 0.$



16. Identify, in the graph to the right, which function is the derivative, and which the original function. (6 points)



f(x) is original

g(x) = derivative = f'(x)