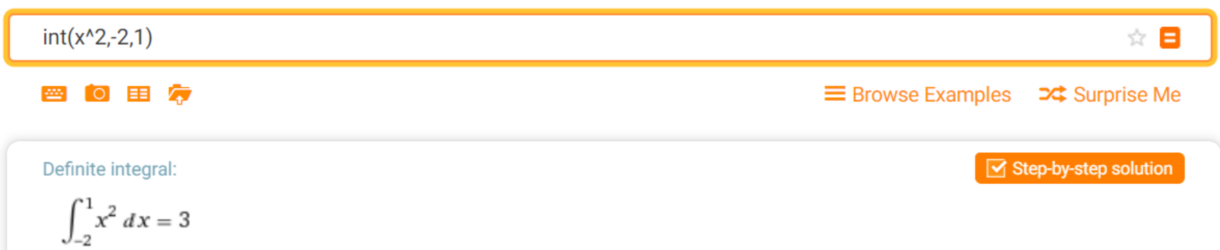
MTH 261, Continuous Probability Distributions Mini-Project, Spring 2019

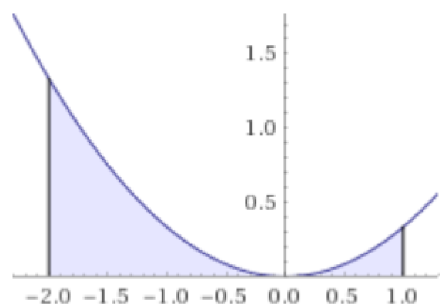
Instructions: Follow the steps shown in the example below. This project uses technology freely available online to explore various continuous probability distributions. Use the online technology (or perform the calculations by hand if you wish) to complete the problems. Include any graphs, and responses to questions, and then submit your file to the dropbox in Blackboard.

For each of the distributions below, you will be asked a series of questions. First, the probability distribution has the form $f(x) = kx^2$ on the interval $[-2,1]$. Recall that for any probability distribution $f(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx = 1$. Thus $\int_{-2}^{1} kx^2 dx = k\int_{-2}^{1} x^2 dx$.

Using the website Wolfram Alpha (https://www.wolframalpha.com/), we find:



Since the integral $\int_{-2}^{1} x^2 dx = 3$, then $k\int_{-2}^{1} x^2 dx = 3k$, and for this to be a probability distribution, $3k = 1$. Thus we know then that $f(x) = \frac{1}{3}x^2$ is a probability density function on the interval $[-2,1]$.



If we change the function or the interval the constant will change, so for each of the problems below, we first need to find the value of k. For each case, include a graph of the function.

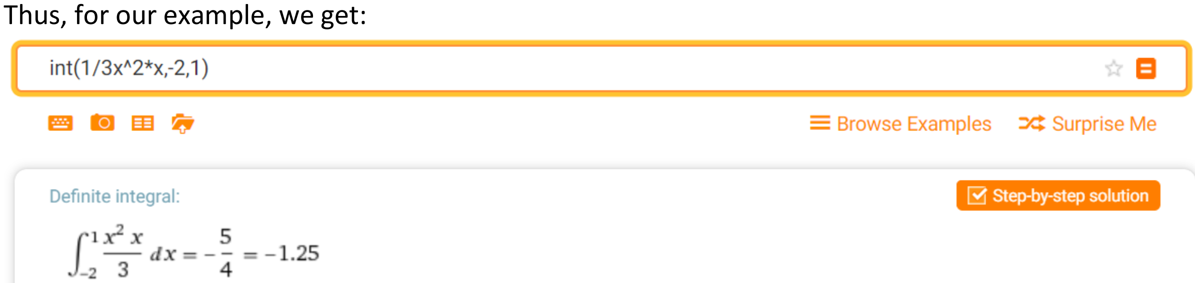
Following that, you will be asked to calculate some probabilities for each function. Do this after finding the value of k.

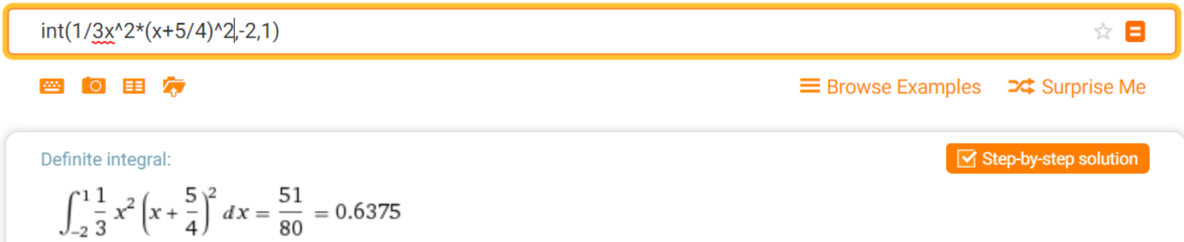
For instance, $P(x < 0) = \int_{-2}^{0} \frac{1}{3}x^2 dx = \frac{8}{9}$ according to Wolfram Alpha. And $P\left(-\frac{1}{2} \leq x < \frac{1}{2}\right) = \int_{-1/2}^{1/2} \frac{1}{3}x^2 dx = \frac{1}{36}$.

Finally, we want to find the mean and standard deviation.

The mean $\mu = \int_a^b xf(x)dx$ and the standard deviation $\sigma = \sqrt{\int_a^b (x-\mu)^2 f(x)dx}$.

Thus, for our example, we get:

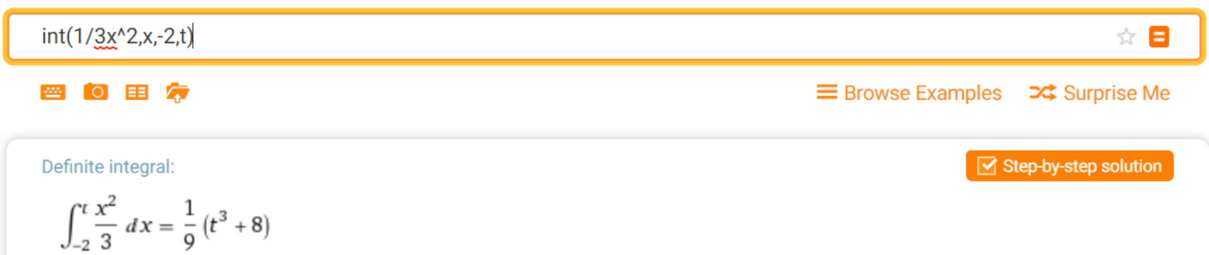


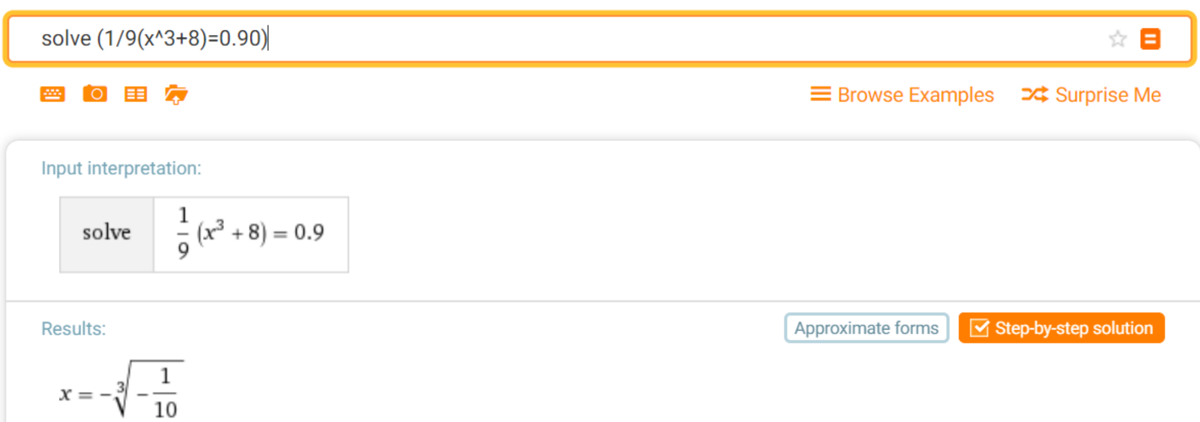


Thus, the mean $\mu = -1.25$ and $\sigma = \sqrt{\frac{51}{80}} \approx 0.7984$.

Lastly, we want to find the indicated percentile value. For us, we'd like to find the value of x which marks the 90th percentile.

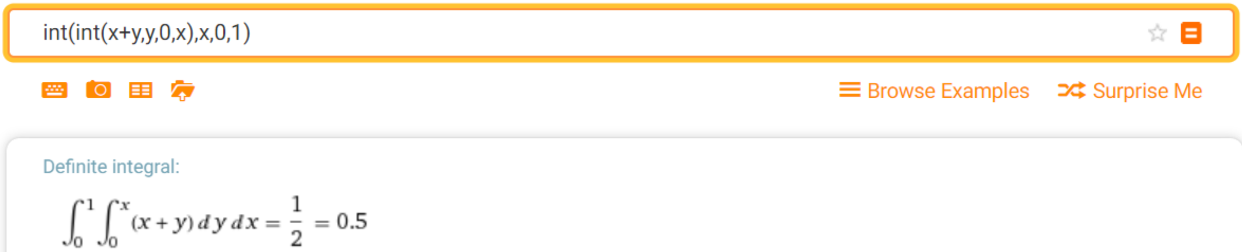
To find the 90th percentile, we calculate $\int_{-2}^{t} \frac{1}{3} x^2 dx = 0.90$. We need the value of t that makes this true. Integrating gives us $\frac{1}{9}(t^3 + 8) = 0.90$.





You will get solutions in various forms. In this case, $t = -\sqrt[3]{-\frac{1}{10}} \approx 0.465158$ …. If Wolfram Alpha gives you any errors, sometimes just rerunning the same input will work on the second try.

Use this model to answer the questions that follow. There is one two-variable problem at the end. It has fewer parts. Integrate in two variables by embedding integrations. The example below shows $f(x, y) = k(x + y), 0 \le x \le 1, 0 \le y \le x$. And recall that this means $k = 2$.



1. $f(x) = kx^2, [-1,1]$
   a. Find k so that $f(x)$ is a probability density function on the given interval.
   b. Provide a sketch of the probability density function on this interval.
   c. Find $P\left(x > \frac{1}{2}\right)$
   d. Find, $P\left(0 \leq x < \frac{1}{2}\right)$
   e. Find the mean μ and standard deviation σ for this distribution.
   f. Find the value of the 70^{th} percentile.

2. $f(x) = \frac{k}{x}, [1,3]$
   a. Find k so that $f(x)$ is a probability density function on the given interval.
   b. Provide a sketch of the probability density function on this interval.
   c. Find $P\left(x > \frac{3}{2}\right)$
   d. Find, $P\left(2 \leq x < \frac{5}{2}\right)$
   e. Find the mean μ and standard deviation σ for this distribution.
   f. Find the value of the 40^{th} percentile.

3. $f(x) = ke^x, [0,2]$
   a. Find k so that $f(x)$ is a probability density function on the given interval.
   b. Provide a sketch of the probability density function on this interval.
   c. Find $P(x < 1)$
   d. Find, $P\left(1 \leq x < \frac{5}{4}\right)$
   e. Find the mean μ and standard deviation σ for this distribution.
   f. Find the value of the 10^{th} percentile.

4. $f(x,y) = kxy, \;\; 0 \leq y \leq x+1, \;\; 0 \leq x \leq 3$
   a. Find k so that $f(x,y)$ is a probability density function on the given region.
   b. Provide a sketch of the region.
   c. Find $P(x < 1)$
   d. Find, $P(x > 2, y \leq 1)$