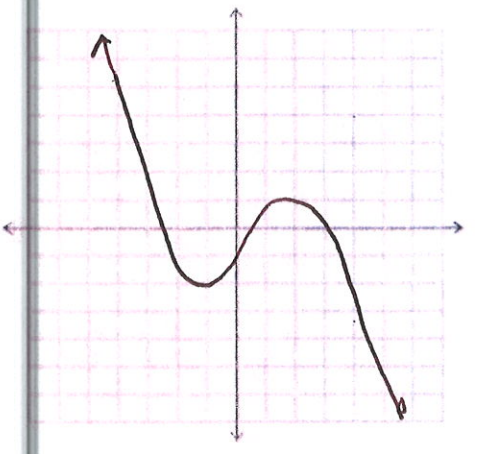
KEY

MTH 161, Practice Exam #3, Spring 2019

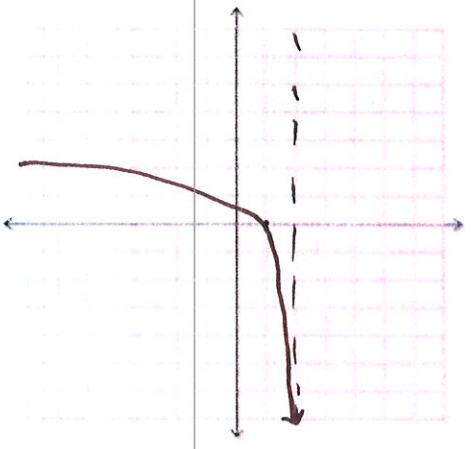
1. Find the function f that is finally graphed after each of the following transformations is applied to $y = 2^x$.
   a. Reflect about the x-axis. -2^x
   b. Shift down one unit. $-2^x - 1$
   $f(x) =$ $-2^x - 1$

2. Sketch a graph of a function which is not one-to-one.



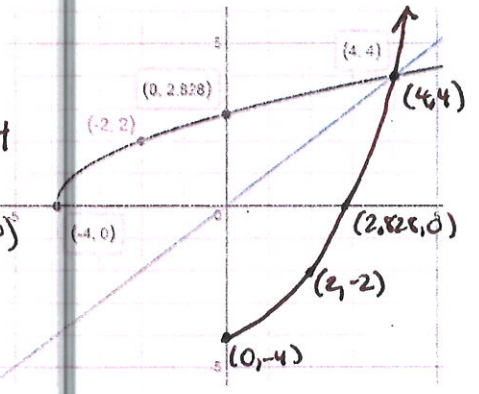
3. A student writes that $(\ln 2)^3 = 3 \ln 2$ by the power property of logarithms. Explain why this is incorrect.

4. Consider the function $f(x) = \ln(2 - x)$. Sketch an accurate graph of the function on graph below.



   a. State and label intercepts and asymptotes.
   $(1,0)$ $(0, \ln 2)$ $x = 2$
   b. The function is one-to-one. How can you tell from the graph? passes horizontal line test
   c. Algebraically find the inverse $f^{-1}(x)$.
   $y = f^{-1}(x) = -e^x + 2$

5. Use the graph below to fill in the function values.



   a. $f(0), f(-2), f^{-1}(2), f^{-1}(4)$
   $f(0) = 2.828$ $f(-2) = 2$ $f^{-1}(2) = -2$ $f^{-1}(4) = 4$
   b. Find the domain and range. How does this relate to the domain and range of $f^{-1}(x)$?
   f: D $[-4, \infty)$, R: $[0, \infty)$ f^{-1}: flip D: $[0, \infty)$, R $[-4, \infty)$
   c. Sketch an accurate graph of f^{-1} on the same graph as f using specific coordinates.

6. Use the properties of logarithms to write the expression $\ln x + \frac{1}{2}\ln(4 - x) - \ln 9$ as a single log expression. Show all intermediate steps.
$\ln\left[\frac{x\sqrt{4-x}}{9}\right]$

7. Use properties of logs to expand the expression in terms of logs of the variables and simplify. Show all steps.

   a. $\ln\left(\frac{\sqrt[4]{x}}{8y}\right)$ $\frac{1}{4}\ln x - \ln 8 - \ln y$
   b. $\log_2(8x^3)$ $\log_2 8 + 3\log_2 x$

8. Use the change of base formula to rewrite the expression $\log_2 0.47$ in terms of natural log.
$\frac{\ln 0.47}{\ln 2}$

9. An initial investment of \$50,000 grows at an annual interest rate of 7% compounded continuously. Use the model $A = Pe^{rt}$ to calculate how long it will take to triple. Show your calculation and express your answer as an exact expression.

15.7 years $\approx \frac{\ln 3}{0.07} = t$

10. Find the exact value of the expression.
    a. $3\ln(e^2)$ 6
    b. $\log(10^{14}) - 10^{3\log 2}$ 6

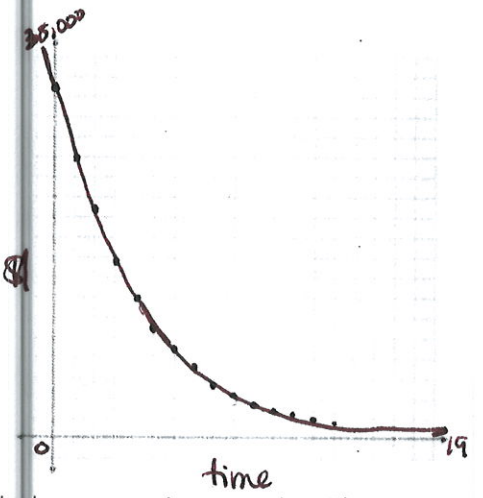
11. Solve the following expressions algebraically. Give exact values.
    a. $2^{3x} = 128$ $x = 7/3$
    b. $\frac{1}{3}\ln(1-x) = 10$ $x = 1 - e^{30}$
    c. $3 - 5e^{3t} = 10$ no solution
    d. $\ln x + \ln(x+2) = 3$ $x = \frac{-2+\sqrt{4+4e^3}}{2} \approx 3.59$

12. In a group project in learning theory, a mathematical model for the percent P of correct responses after n trials was found to be $P(n) = \frac{0.9}{1+e^{-0.1n}}$.
    a. What percent of the responses are correct in the first trial? $\frac{.9}{1+e^{0.1}} \approx .4225$ or 42.25%
    b. After how many trials will 70% of the responses be correct? $n \geq 13$ $n \geq -10\ln(3/7)$

13. A new truck costs \$35,000. The value of the truck after t years is modeled by $V(t) = 35000\left(\frac{4}{5}\right)^t$.
    a. Evaluate the function at the given times.

t	$V(t)$
0	35,000
1	28,000
2	22,400
3	17,920

    b. Sketch the graph by hand in an appropriate window range. Label axes.



    c. Find the value of the truck after 7 years. Show work.
    \$7340
    d. Based on the graph, describe how the value of the truck changes as time goes by. Then complete the statement: As time goes by, value decreases. As $t \to \infty$, $V(t) \to$ 0.

14. Given the graph of $f(x) = \ln x$, then the graph of $y = \ln(x-k)$, $k > 0$, then which of th following is true?
    a. A vertical asymptote at $x = k$.
    b. A horizontal asymptote at $y = k$.
    c. A vertical asymptote at $x = 0$.
    d. A horizontal asymptote at $y = 0$.

15. What logarithmic equation is equivalent to $c^b = a$? $\log_c a = b$

16. What is the first step to solving the equation $2 + \ln(x + 1) = 7$.

Subtract 2 from both sides

17. Which statement is true? Mark all that apply.

a. $2^x \log_2 x = x$
b. $\log_2 2^x = x$
c. $2^{\log_2 x} = x$
d. $\log_2 x = 2^x$

18. The expression $\log x - 3 \log 2$ is equivalent to $\log(x - 8)$ is true or false?

false $\log x - \log 2^3 =$

$\log x - \log 8 =$

$\log\left(\frac{x}{8}\right) \neq \log(x-8)$

MTH 161 Practice Exam #3 Key work

3. Rule is $\ln(M^r) = r \ln M$

not $(\ln M)^r = r \ln M$ and $\ln(M^r) \neq (\ln M)^r$

4c. $y = \ln(2-x)$

$x = \ln(2-y)$

$e^x = 2-y$

$y = 2 - e^x$ or $y = -e^x + 2$

6. $\ln x + \frac{1}{2}\ln(4-x) - \ln 9 = \ln x + \ln\sqrt{4-x} - \ln 9$

$= \ln[x\sqrt{4-x}] - \ln 9 = \ln\left[\frac{x\sqrt{4-x}}{9}\right]$

7a. $\ln\left(\frac{\sqrt[4]{x}}{8y}\right) = \ln\sqrt[4]{x} - \ln 8y = \frac{1}{4}\ln x - \ln 8 - \ln y$

b. $\log_2(8x^3) = \log_2 8 + \log_2 x^3 = \log_2 8 + 3\log_2 x$

9. $A = 50{,}000e^{.07t}$

$3 = e^{.07t} \rightarrow \ln 3 = .07t \rightarrow \frac{\ln 3}{.07} = t \approx 15.7$ years

10. a. $3[2\ln e] = 3[2] = 6$

b. $\log 10^{14} - 10^{3\log 2} = 14 - 10^{\log 2^3} = 14 - 8 = 6$

11a. $2^{3x} = 2^7 \rightarrow 3x = 7 \rightarrow x = 7/3$

b. $\ln(1-x)^{1/3} = 10 \rightarrow \sqrt[3]{1-x} = e^{10} \rightarrow 1-x = e^{30} \rightarrow x = 1 - e^{30}$

c. $3 - 5e^{3t} = 10 \rightarrow -5e^{3t} = 7 \rightarrow e^{3t} = -7/5$ no solution

d. $\ln x + \ln(x+2) = 3 \rightarrow \ln[x(x+2)] = 3 \rightarrow x(x+2) = e^3 \rightarrow x^2 + 2x - e^3 = 0$

$x = \frac{-2 \pm \sqrt{4+4e^3}}{2} \approx 3.591899$, no neg. solution

12b. $0.7 = \frac{0.9}{1+e^{-0.1n}} \rightarrow 1+e^{-0.1n} = \frac{0.9}{0.7} \rightarrow$

$e^{-0.1n} = \frac{9}{7} - 1 \rightarrow -0.1n = \ln\left(\frac{9}{7} - 1\right) \rightarrow n = -10\ln\left(\frac{9}{7} - 1\right)$

$= -10\ln\left(\frac{2}{7}\right)$

$n \approx 12.528$

13c. $35000\left(\frac{4}{5}\right)^7 = 7340$