1/14/2019
MTH 154

We are going to first look at logic and set notation basics because they tend to appear in statistics when we do probability problems.

Set Notation Basics

Finite lists of discrete elements can be listed in curly brackets. The letters in the name Kamala would be $\{A, K, L, M\}$. Elements used more than once are listed only once.

Lists of discrete elements that follow a pattern can also be listed this way, such as all even numbers would look like $\{\dots, -4, -2, 0, 2, 4, \dots\}$ with the ellipses (…) to indicate that the pattern continues.

Continuous sets of real numbers can be stated as equations in a slightly different format. Infinite sets like the set of even numbers that follows a formula can also be listed this way. For instance, the set of even numbers can be written as $\{x|x = 2n, n \in Z\}$. This set notation is read "The set of elements x, such that x can be expressed in the form $2n$ where n is in the set Z (which is the set of all integers).

Breaking this down: {} indicates that this is a set

- x is the name we are using for general elements in the set, which will appear in the formula
- | is read "such that". What follows this is the set of conditions the elements of the set must satisfy
- $x = 2n$ is condition on elements in the sets. $2n$ is a typical expression for even numbers while $2n + 1$ is used for odd numbers.
- ∈ is read as "is in" or "in an element of" the set that follows
- Z is an abbreviation for the set of integers (positive and negative whole numbers). R is the set of real numbers and is another common set we might draw from.

This entire format is sometimes referred to as set-builder notation.

If we use real numbers, our sets might also end up being written in interval notation. For example, $\{x|x^2 \leq 1, x \in R\}$ is the set of real numbers that satisfy the equation $x^2 \leq 1$, if we think about this a bit, we find that the elements are real numbers between -1 and 1, or $-1 \leq x \leq 1$. In interval notation we can write this as $[-1,1]$. (We won't see a lot of this. We'll mostly be looking at finite sets.)

Sometimes you'll also see statements or other explanations as the conditions. Such as $\{x|x \textit{ is a positve integer less than } 20 \textit{ that is divisible by } 3\}$, which is equivalent to $\{3,6,9,12,15,18\}$.

One task we have is to translate these different forms of set notation into another form. Such as translating set-builder notation into a list of elements. Problem #2 on the homework gives you some practice with this.

Another thing we want to do is do operations on sets and compare sets.
One way of comparing sets is to talk about whether one set contains another. At this point, we need to name the sets so we can refer to them easily.

$$A = \{6,12,18\}, \qquad B = \{3, 6, 9, 12, 15, 18\}$$

Consider the two sets A and B. Since all the elements of A also are elements of B, we can say $A \subset B$ or "A is a subset of B". Note that this is different than when we refer to a specific element in B, such as $6 \in B$, which just means 6 is an element of B. If we want 6 to be in a subset by itself we'd have to write $\{6\} \subset B$. This can be a little confusing, but think of sets like bags with stuff in them. If the bag has only one thing in it, then that is like a set with only one element like {6}, but we are still comparing this bag to that bag, which is a comparison of sets. The bags might also have more than one thing in them. Or they might be empty. The notation for that is {} an empty set of brackets, or ∅ the empty set. On the other hand, if we open the bag and look at the things inside, that is like looking at elements, not whole bags. Problem #3 on the homework gives you some practice with this.

Operations on sets include intersection, union and complements.

- ∪ represents the union of sets, $A \cup B$ can be read as "the set of all elements that are in either set A **or** set B".
- ∩ represents the intersection of sets, $A \cap B$ can be read as "the set of all elements that are in both A **and** B".
- The complement is the "opposite" notation, all those elements not in the set. Usually, this is in comparison to some Universal set. The notation for complements are not standard. I'll use A' to indicate "not-A" or "those elements not in A", but if you use other resources, you'll see other notations.
- We can also talk about the number of elements in the set as either $|A|$ or $n(A)$. Both of just mean for us to count the number of elements. If the set is finite, then we'll get a number. Some sets will be infinite, but we will not be dealing with many of those.

Let's put this together with an example.

Consider the universal set U is the set of all positive numbers less then 12. And $A = \{1, 4, 7, 9, 11\}$ and $B = \{2, 5, 8, 11\}$.

$$A \cup B = \{1, 2, 4, 5, 7, 8, 9, 11\}$$

This is all the elements that appear in either A or B, and we don't repeat in the list elements that appear more than once.

$$A \cap B = \{11\}$$

This is all the elements that appear in both sets. The only one that appears in both sets is 11.

$$A' = \{2, 3, 5, 6, 8, 10\}$$

The Universal set is $\{\mathbf{1}, 2,3, \mathbf{4}, 5,6, \mathbf{7}, 8, \mathbf{9}, 10, \mathbf{11}\}$, and you can see the elements in A in boldface. All the other elements are not in A, and so are in the set "not-A" or A'.

$$n(A) = 5$$

The number of elements in A is 5.
Problems #1 and #4 give you practice with this.

Some other operations on sets include $A - B$ which is "the set of all elements in A that does not also appear in B". In our example above, $A - B = \{1, 4,7,9\}$. Since 11 is in B, it has to be removed from A.

Another one is ×, which makes ordered pairs of the elements in the set, like coordinate points on a graph. $A \times B$ is a list of pairs where the first element comes from A and the second from B. And example would be $(1,2)$.

The complete list looks like this:
$\{(1,2),(1,5),(1,8),(1,11),(4,2),(4,5),(4,8),(4,11),(7,2),(7,5),(7,8),(7,11),(9,2),(9,5),(9,8),(9,11),(11,2),(11,5),(11,8),(11,11)\}$

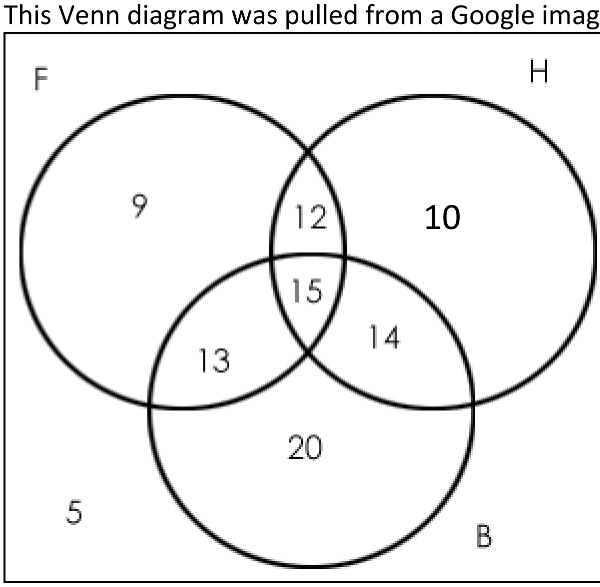
One way of visualizing this list is in a table. Problem #7 is some practice with this.

B↓, A→	1	4	7	9	11
2	(1,2)	(4,2)	(7,2)	(9,2)	(11,2)
5	(1,5)	(4,5)	(7,5)	(9,5)	(11,5)
8	(1,8)	(4,8)	(7,8)	(9,8)	(11,8)
11	(1,11)	(4,11)	(7,11)	(9,11)	(11,11)

Venn diagrams can be a way of visualizing what is going on with basic operations like union, intersection, complements, and subsets.

There is a handout on sets and Venn diagrams in Blackboard that you can look at for more examples.

This Venn diagram was pulled from a Google image search.



Let's understand what it means. The big box on the outside is the universal set. Everything inside it is $\{5,9,10,12,13,14,15,20\}$. There are three sets F, H and B. $F = \{9,12,13,15\}$, $H = \{10,12,14,15\}$, and $B = \{13,14,15,20\}$.

$$F \cup B = \{9,12,13,15,14,20\}$$
$$H \cap B = \{14,15\}$$
$$H' = \{5,9,13,20\}$$

We could even ask $F \cap H \cap B$ the set of elements in all three sets, and that would just be $\{15\}$. Problem #6 will give you some practice working with these. In the homework, shade in the part of the graph that corresponds to the described set.

This is all the material covered on the first quiz.

There are resources in the course that can help with this material. There is a handout on Venn Diagrams and set notation with more worked examples. There is a handout on statistical symbols which includes many of the set notation symbols here. On the Archive set, there is also a link to a video playlist that covers both of these components.

Our next topic is logical notation, which is related to set notation. For the most part, we'll be focused on learning to read the notation. And starting to think about using Excel.