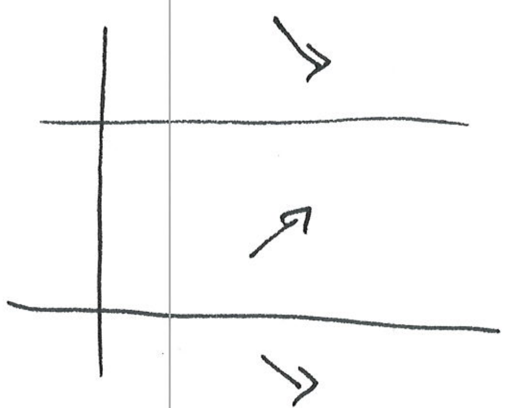
Fish Harvesting

A mathematician at a fish hatchery has been using the differential equation $\frac{dP}{dt} = 2P\left(1 - \frac{P}{25}\right)$ as a model for predicting the number of fish that a hatchery can expect to find in their pond.

1. Use an autonomous derivative graph, a phase line, and a slope field to analyze what this differential equation predicts for future fish populations for a range of initial conditions. Present all three of these representations and describe in a few sentences how to interpret them.

$P = 0, \; P = 25 \qquad \frac{dP}{dt} = 0$



$P = 25$ stable

$P = 0$ unstable

descriptions will vary

2. Recently, the hatchery was bought out by fish.net and the new owners are planning to allow the public to catch fish at the hatchery (for a fee of course). This means that the previous differential equation used to predict future fish populations needs to be modified to reflect this new plan. For the sake of simplicity, assume that this new plan can be taken into consideration by including a constant, annual harvesting rate k into the previous differential equation. Below are two modifications to the differential equation that may account for the new plan. Do you agree with either of these? If yes, explain why. If no, create your own modification and explain your reasoning.

(a) $\frac{dP}{dt} = 2P\left(1 - \frac{P}{25}\right) - kP$

(b) $\frac{dP}{dt} = 2P\left(1 - \frac{P-k}{25}\right)$

$\frac{dP}{dt} = 0$

$0 = 2P\left(1 - \frac{P}{25}\right) - kP$

$kP = 2P\left(1 - \frac{P}{25}\right)$

$0 = P\left[2\left(1 - \frac{P}{25}\right) - k\right]$

$= P\left[2 - \frac{2P}{25} - k\right]$

$= P\left[(2-k) - \frac{2P}{25}\right]$

$P = 0$ remains the same

$2 - k = \frac{2P}{25}$

$50 - 25k = 2P$

$P = 25 - \frac{25k}{2}$

varies level of carrying capacity

$\frac{dP}{dt} = 0$

$2P = 0$ remains the same

$0 = 1 - \frac{P-k}{25}$

$25 = P - k$

$25 + k = P$

also modifies the level of the carrying capacity

I think the first version is more likely in this scenario. The second appears to reflect additional feeding rather than additional harvesting.
KP doesnot reflect a constant harvest.

3. Your team of consultants settled on $\frac{dP}{dt} = 2P\left(1 - \frac{P}{25}\right) - k$ to model the new fishing plan. Analyze the effect of different choices for the value of k on the fish population. Synthesize your analysis in a **one page** report for the new owners that illustrates the implications that various choices of k will have on future fish populations. Your report may include one or more graphical representations but must communicate the effect of different k values in a concise way.

old population was stable at $P = 25$.

$2P - \frac{2P^2}{25} - k = 0 \Rightarrow 50P - 2P^2 - 25k = 0$

$P = \frac{-2 \pm \sqrt{4 - 4\left(\frac{2}{25}\right)(-k)}}{2\left(-\frac{2}{25}\right)} = \frac{-50 \pm \sqrt{2500 - 4(-2)(-25k)}}{-4}$

$\frac{-50 \pm \sqrt{2500 - 200k}}{-4}$

$\frac{-50}{-4} \pm \frac{\sqrt{25-2k}}{-4}\ (10)$

$\frac{25}{2} \pm \frac{5\sqrt{25-2k}}{2}$

equilibria get closer together if k positive
Centered at 12.5
if k is < 12.5 the carrying capacity will remain above the threshold, but the closer together they get, the greater the threat of overfishing.