

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use Euler's Method with the indicated step size to find an estimate of the solution at the indicated point.

$$y' = x + y^2, y(0) = 1, \Delta x = 0.2, y(1) = ?$$

n	x_n	y_n	m_n	x_{n+1}
0	0	1	$1 + 1^2 = 1$	$1 + 1(0.2) = 1.2$
1	0.2	1.2	$1.2 + 1.2^2 = 1.64$	$1.2 + 1.64(0.2) = 1.528$
2	0.4	1.528	$1.528 + 1.528^2 = 2.73$	$1.528 + 2.73(0.2) = 2.0749568$
3	0.6	2.075	$2.075 + 2.075^2 = 4.905$	$2.075 + 4.905(0.2) = 3.056$
4	0.8	3.056	$3.056 + 3.056^2 = 10.14$	$3.056 + 10.14(0.2) = 5.0839$
5	1.0	5.084		

$$y(1) \approx 5.0839$$

2. Show that $y = -t \cos t - t$ is a solution of the initial value problem $t \frac{dy}{dt} = y + t^2 \sin t$, $y(\pi) = 0$.

$$\begin{aligned} y' &= -\cos t + t \sin t - 1 \\ t y' &= -t \cos t + t^2 \sin t - t \\ &= (-t \cos t - t) + t^2 \sin t \\ &= y + t^2 \sin t \quad \checkmark \end{aligned}$$

$$y = -\pi \cos \pi - \pi = \pi - \pi = 0 \quad \checkmark$$