MTH 291, Quiz #11, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the system of linear equations. (Your solutions should be expressed in real-valued expressions.)

$$\vec{x}'(t) = \begin{pmatrix} 1 & -8 \\ 1 & -3 \end{pmatrix}\vec{x}, \vec{x}(0) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

$(1-\lambda)(-3-\lambda)+8=0$

$\lambda^2+2\lambda-3+8=0$

$\lambda^2+2\lambda+5=0$

$\lambda = \frac{-2\pm\sqrt{4-4(5)}}{2} =$

$\frac{-2\pm 4i}{2} = -1\pm 2i$

$\begin{bmatrix} 1-(-1+2i) & -8 \\ 1 & -3-(-1+2i) \end{bmatrix} =$

$\begin{bmatrix} 2-2i & -8 \\ 1 & -2-2i \end{bmatrix}$

$x_1 = (2+2i)x_2$

$x_2 = x_2$

$\begin{bmatrix} 2+2i \\ 1 \end{bmatrix} e^{(-1+2i)t} = e^{-t}\begin{bmatrix} 2+2i \\ 1 \end{bmatrix}(\cos 2t + i\sin 2t)$

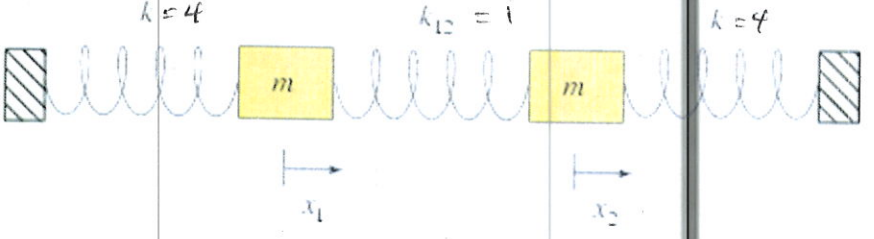
$= e^{-t}\begin{bmatrix} 2\cos 2t + 2i\sin 2t + 2i\cos 2t - 2\sin 2t \\ \cos 2t + i\sin 2t \end{bmatrix} \Rightarrow$

$\vec{x}(t) = c_1 e^{-t}\begin{bmatrix} 2\cos 2t - 2\sin 2t \\ \cos 2t \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 2\sin 2t + 2\cos 2t \\ \sin 2t \end{bmatrix}$

$\begin{pmatrix} -1 \\ 6 \end{pmatrix} = c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\quad c_1 = 6,\ c_2 = -\frac{13}{2}$

$\vec{x}(t) = 6e^{-t}\begin{bmatrix} 2\cos 2t - 2\sin 2t \\ \cos 2t \end{bmatrix} - \frac{13}{2}e^{-t}\begin{bmatrix} 2\sin 2t + 2\cos 2t \\ \sin 2t \end{bmatrix}$

2. Set up a system of equations to model the spring system in the diagram below. Let $k = 4, k_{12} = 1$. You do not need to solve it.



$mx_1'' + (5)x_1 - x_2 = 0$

$mx_2'' + 5x_2 - x_1 = 0$

$x_1'' = -\frac{5}{m}x_1 + \frac{x_2}{m}$

$x_2'' = \frac{1}{m}x_1 - \frac{5}{m}x_2$