MTH 291, Quiz #10, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the system of linear equations. (Your solutions should be expressed in real-valued expressions.)

a. $\begin{cases} \frac{dx}{dt} = 2x + 3y \\ \frac{dy}{dt} = 2x + y \end{cases}$

$A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$

$(2-\lambda)(1-\lambda) - 6 = 0$

$\lambda^2 - 3\lambda + 2 - 6 = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\lambda = 4, \lambda = -1$

$\lambda = 4$

$\begin{bmatrix} 2-4 & 3 \\ 2 & 1-4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}$

$2x_1 = 3x_2$

$x_1 = \frac{3}{2}x_2$ $\quad \vec{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\lambda = -1$

$\begin{bmatrix} 2+1 & 3 \\ 2 & 1+1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$

$x_1 = -x_2$ $\quad \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\vec{x}(t) = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t}$

b. $\begin{cases} \frac{dx}{dt} = 6x - y \\ \frac{dy}{dt} = 5x + 4y \end{cases}$

$\begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}$

$(6-\lambda)(4-\lambda) + 5 = 0$

$\lambda^2 - 10\lambda + 24 + 5 = 0$

$\lambda^2 - 10\lambda + 29 = 0$

$\lambda = \dfrac{10 \pm \sqrt{100 - 116}}{2}$

$= \dfrac{10 \pm 4i}{2} = 5 \pm 2i$

$\lambda = 5 + 2i$

$\begin{bmatrix} 6-5-2i & -1 \\ 5 & 4-5-2i \end{bmatrix} = \begin{bmatrix} 1-2i & -1 \\ 5 & -1-2i \end{bmatrix}$

$5x_1 = (1+2i)x_2$

$x_1 = \frac{1+2i}{5} x_2$ $\quad \vec{v}_1 = \begin{bmatrix} 1+2i \\ 5 \end{bmatrix}$

$\begin{bmatrix} 1+2i \\ 5 \end{bmatrix} e^{(5+2i)t} = e^{5t} \begin{bmatrix} 1+2i \\ 5 \end{bmatrix} (\cos 2t + i \sin 2t)$

$= e^{5t} \begin{bmatrix} \cos 2t + i \sin 2t + 2i \cos 2t - 2 \sin 2t \\ 5 \cos 2t + 5i \sin 2t \end{bmatrix}$

$\vec{x}(t) = c_1 e^{5t} \begin{pmatrix} \cos 2t - \sin 2t \\ 5 \cos 2t \end{pmatrix} +$

$+ c_2 e^{5t} \begin{pmatrix} \sin 2t + 2\cos 2t \\ 5 \sin 2t \end{pmatrix}$