MTH 277, Quiz #8, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Draw the level curves of the function $f(x,y) = x^2 + 2xy$. Use the contour curves to sketch the gradient of the function.

$\nabla f = \langle 2x+2y, 2x \rangle$

$2x+2y=0 \quad 2x=0$

$2x=-2y \quad x=0$

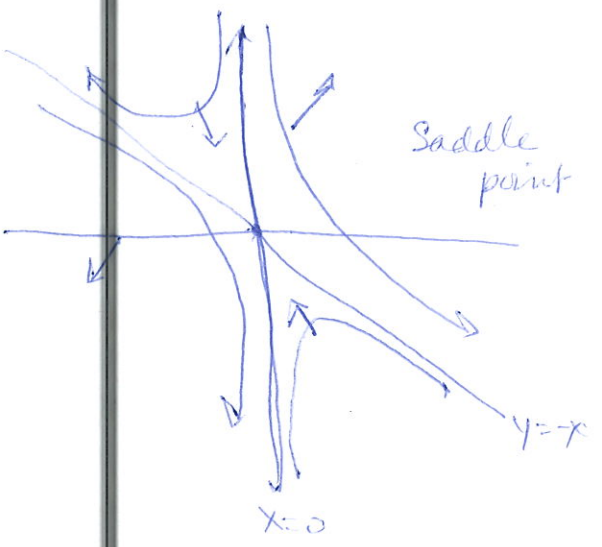
$x=-y$

(x,y)	∇f
$(1,4)$	$\langle 10, 2 \rangle$
$(-1,1)$	$\langle 0, -2 \rangle$
$(1,-4)$	$\langle -6, 2 \rangle$
$(-4,0)$	$\langle -8, -8 \rangle$

$c = x^2 + 2xy$

$\frac{c-x^2}{2x} = y$

$c=1, \; c=-1$



2. Find the potential function for the conservative vector field $\vec{F}(x,y,z) = \tan(y+z)\,\hat{\imath} + x\sec^2(y+z)\,\hat{\jmath} + x\sec^2(y+z)\,\hat{k}$.

$\int \tan(y+z)\,dx = x\tan(y+z) + f(y,z)$

$\int x\sec^2(y+z)\,dy = x\tan(y+z) + g(x,z)$

$\int x\sec^2(y+z)\,dz = x\tan(y+z) + h(x,y)$

$\varphi(x,y,z) = x\tan(y+z) + K$

3. Integrate $\int_0^2 \int_0^x (1 + 2x + 2y)\,dy\,dx$.

$\int_0^2 y + 2xy + y^2 \Big|_0^x dx = \int_0^2 x + 2x^2 + x^2 - 0\,dx =$

$\int_0^2 x + 3x^2\,dx = \frac{1}{2}x^2 + x^3 \Big|_0^2 = \frac{1}{2}(4) + 2^3 - 0 = 2 + 8 = \boxed{10}$