MTH 277, Quiz #2, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the domain and range of the function $f(x,y) = \arcsin\left(\frac{x}{y}\right)$. Use appropriate notation.

$-1 \le \frac{x}{y} \le 1$

$-y \le x \le y$

$D: \{(x,y) \mid -y \le x \le y\}$

$R: [-\pi/2, \pi/2]$

2. Find the equation of the plane containing the points $(1,3,-7), (0,-1,4), (3,2,2)$.

$\vec{v} = \langle 1, 4, -11 \rangle$ $\vec{w} = \langle -3, -3, 2 \rangle$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -11 \\ -3 & -3 & 2 \end{vmatrix} = (8-33)\hat{i} - (2-33)\hat{j} + (-3+12)\hat{k}$$

$$= -25\hat{i} + 31\hat{j} + 9\hat{k}$$

$$-25(x-1) + 31(y-3) + 9(z+7) = 0$$

3. Identify the quadric surface $x^2 - y^2 + z = 0$, and then convert the equation to cylindrical and spherical coordinates.

$z = y^2 - x^2$

hyperbolic paraboloid

(increasing on y-axis, decreasing on x)

Cylindrical

$z = r^2\sin^2\theta - r^2\cos^2\theta$

$z = -r^2\cos 2\theta$

Spherical

$\rho\cos\varphi = \rho^2\sin^2\theta\sin^2\varphi - \rho^2\cos^2\theta\sin^2\varphi$

$\rho\cos\varphi = \rho^2\sin^2\varphi(-2\cos 2\theta)$

$$-\tfrac{1}{2}\sec 2\theta\cot\varphi\csc\varphi = \rho$$

4. Identify the surface given by $\vec{r}(u,v) = 3\cos v \cos u\,\hat{\imath} + 3\cos v \sin u\,\hat{\jmath} + 5\sin v\,\hat{k}$.

$$\frac{x^2}{3^2} + \frac{y^2}{3^2} + \frac{z^2}{5^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{25} = 1$$ ellipsoid