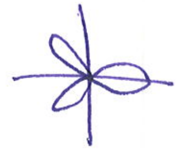
MTH 277, Quiz #21, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the center of mass for region bounded by $r = 2\cos 3\theta$ for $\theta \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$ and $\rho = kr$.



$$M = \iint_R \rho \, dA = k\int_{-\pi/6}^{\pi/6}\int_0^{2\cos 3\theta} r \cdot r \, dr \, d\theta = k\int_{-\pi/6}^{\pi/6} \frac{1}{3}r^3 \Big|_0^{2\cos 3\theta} d\theta =$$

$$\frac{k}{3}\int_{-\pi/6}^{\pi/6} 8\cos^3 3\theta \, d\theta = \frac{8k}{3}\int_{-\pi/6}^{\pi/6} \cos 3\theta(1-\sin^2 3\theta)\,d\theta =$$

$u = \sin 3\theta$
$du = 3\cos 3\theta \, d\theta$

$$\frac{8k}{9}\left[\sin 3\theta - \frac{1}{3}\sin^3 3\theta\right|_{-\pi/6}^{\pi/6} = \frac{8k}{9}\left[1 - \frac{1}{3}(1)^3 - (-1) + \frac{1}{3}(-1)^3\right]$$

$$= \frac{8k}{9}\left[2 - \frac{2}{3}\right] = \frac{8k}{9}\left[\frac{4}{3}\right] = \frac{32k}{27}$$

$$M_x = k\int_{-\pi/6}^{\pi/6}\int_0^{2\cos 3\theta} \underset{(r\sin\theta)}{y} \, r \cdot r \, dr \, d\theta = k\int_{-\pi/6}^{\pi/6}\int_0^{2\cos 3\theta} r^3 \sin\theta \, dr \, d\theta =$$

$$\frac{k}{4}\int_{-\pi/6}^{\pi/6} r^4 \Big|_0^{2\cos 3\theta} \sin\theta \, d\theta = 4k\int_{-\pi/6}^{\pi/6} \cos^4 3\theta \cdot \sin\theta \, d\theta = 0 \cdot 4k = 0$$

$$M_y = k\int_{-\pi/6}^{\pi/6}\int_0^{2\cos 3\theta} \underset{(r\cos\theta)}{x} \cdot r \cdot r \, dr \, d\theta = k\int_{-\pi/6}^{\pi/6}\int_0^{2\cos 3\theta} r^3 \cos\theta \, dr \, d\theta =$$

$$\frac{k}{4}\int_{-\pi/6}^{\pi/6} r^4 \Big|_0^{2\cos 3\theta} \cos\theta \, d\theta = 4k\int_{-\pi/6}^{\pi/6} \cos^4 3\theta \cos\theta \, d\theta = \frac{1944}{5005} \cdot 4k = \frac{7776}{5005}k$$

$$\bar{x} = \frac{M_y}{M} = \frac{7776\cancel{k}}{5005} \cdot \frac{27}{32\cancel{k}} = \frac{6561}{5005} \approx 1.31 \qquad (\bar{x}, \bar{y}) = (1.31, 0)$$

$$\bar{y} = \frac{M_x}{M} = \frac{0}{\frac{32k}{27}} = 0$$