MTH 277, Quiz #19, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Sketch the gradient field for $f(x,y) = x^2 + y^2 + 2x - 6y + 6$ and use it to characterize any critical points. Verify the solution with the second partials test.

$\nabla f = \langle 2x+2,\ 2y-6 \rangle$

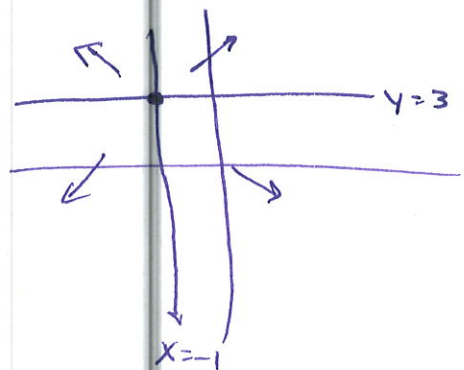
$2x+2=0 \qquad 2y-6=0$

$x=-1 \qquad y=3$

critical point $(-1,3)$

minimum

∇f	(x,y)
$\langle 2,-6\rangle$	$(0,0)$
$\langle 2,2\rangle$	$(0,4)$
$\langle -2,-6\rangle$	$(-2,0)$
$\langle -2,2\rangle$	$(-2,4)$



$f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = 0$

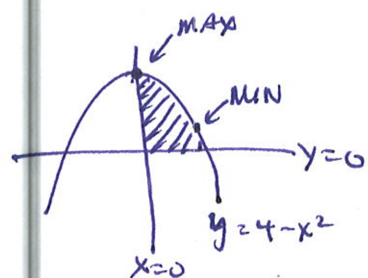
$D = 2(2) - 0 = 4 > 0,\ f_{xx} > 0 \ \cup$ minimum

2. Find the absolute extrema for $f(x,y) = 2x + 2y^2 - 4xy$ on the region bounded by $x = 0, y = 0, y = 4 - x^2$.

$\nabla f = \langle 2-4y,\ 4y-4x \rangle$

$4y = 2 \qquad y = x$

$y = \frac{1}{2}$



pts to check

$(\frac{1}{2},\frac{1}{2})$

$(0,0)$

$(1.625, 1.36)$

$(2,0)$

$(0,4)$

$f(0,y) = 2y^2 \rightarrow f'(y) = 4y \rightarrow y=0$

$f(x,0) = 2x \rightarrow f'(x) = 2 \neq 0$

$f(x,4-x^2) = 2x + 2(4-x^2)^2 - 4x(4-x^2)$

$= 2x + 2(16 - 8x^2 + x^4) - 16x + 4x^3$

$= 2x + 32 - 16x^2 + 2x^4 - 16x + 4x^3$

$= 32 - 14x - 16x^2 + 4x^3 + 2x^4$

$\rightarrow f'(x) = -14 - 32x + 12x^2 + 8x^3$

$x \approx \cancel{-2.73}, \cancel{-.3945}, 1.625$

$f(\frac{1}{2},\frac{1}{2}) = \frac{1}{2}$

$f(0,0) = 0$

$f(1.625, 1.36) = -1.891$ ABS min

$f(2,0) = 4$

$f(0,4) = 32$ ABS Max