MTH 277, Quiz #16, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the surface integral $\int_S \int xy\,dS$ for $S: z = 3 - x - y$ in the first octant.

$G = 3 - x - y - z$ $\quad$ $3 - x = y$

$\nabla G = \langle -1, -1, -1 \rangle$

$\|\nabla G\| = \sqrt{3}$

$$\int_0^3 \int_0^{3-x} xy\sqrt{3}\,dy\,dx =$$

$$\sqrt{3}\int_0^3 \frac{1}{2}y^2 x\Big|_0^{3-x} dx = \frac{\sqrt{3}}{2}\int_0^3 x(3-x)^2 dx = \frac{\sqrt{3}}{2}\int_0^3 x(9-6x+x^2)dx = \frac{\sqrt{3}}{2}\int 9x - 6x^2 + x^3 dx$$

$$= \frac{\sqrt{3}}{2}\left[\frac{9}{2}x^2 - 2x^3 + \frac{x^4}{4}\right]_0^3 = \frac{\sqrt{3}}{2}\left[\frac{81}{2} - 54 + \frac{81}{4}\right] = \boxed{\frac{27\sqrt{3}}{8}}$$

2. Evaluate $\int_S \int \vec{F} \cdot \vec{N}\,dS$ where $\vec{N}$ is the upward normal for $\vec{F}(x, y, z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, where $S: z = 1 - x^2 - y^2, z \geq 0$.

$G = -1 + x^2 + y^2 + z$

$\nabla G = \langle 2x, 2y, 1 \rangle$

$F \cdot \nabla G = \langle x, y, z \rangle \cdot \langle 2x, 2y, 1 \rangle =$

$2x^2 + 2y^2 + z$

$2x^2 + 2y^2 + (1 - x^2 - y^2) = 2x^2 + 2y^2 + 1 - x^2 - y^2 = x^2 + y^2 + 1$

$= r^2 + 1$

$$\int_0^{2\pi}\int_0^1 (r^2+1)r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 r^3 + r\,dr\,d\theta = \int_0^{2\pi} \frac{1}{4}r^4 + \frac{1}{2}r^2\Big|_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{3}{4}d\theta = 2\pi \cdot \frac{3}{4} = \boxed{\frac{3\pi}{2}}$$