MTH 277, Quiz #14, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the surface area of the function $z = 16 - x^2 - y^2$ in the first octant.

$F = 16 - x^2 - y^2 - z$

$u = 4r^2 + 1 \quad \begin{cases} 4 \to 65 \\ 0 \to 1 \end{cases}$

$\nabla F = \langle -2x, -2y, -1 \rangle$

$du = 8r\,dr$

$\|\nabla F\| = \sqrt{4x^2 + 4y^2 + 1}$

$\frac{1}{8}du = r\,dr$

$S = \iint_R \sqrt{4x^2 + 4y^2 + 1}$

$= \int_0^{\pi/2}\int_0^4 \sqrt{4r^2+1}\; r\,dr\,d\theta = \int_0^{\pi/2} \frac{1}{12}(65^{3/2} - 1)\,d\theta = \boxed{\frac{\pi}{24}(65^{3/2} - 1)}$

$\frac{1}{8}\int u^{1/2}\,du \Rightarrow \frac{1}{\cancel{48}}\,\frac{\cancel{2}}{3}\,u^{3/2}\Big|_1^{65}$

$\frac{1}{12}(65^{3/2} - 1)$

2. Find the surface area of the surface $\vec{r}(u, v) = 4\cos u\,\hat{\imath} + 4\sin u\,\hat{\jmath} + v\hat{k}$ for $0 \le u \le \pi, 0 \le v \le 2$.

$\vec{r}_u = -4\sin u\,\hat{\imath} + 4\cos u\,\hat{\jmath} + 0\hat{k}$

$\vec{r}_v = 0\,\hat{\imath} + 0\,\hat{\jmath} + 1\hat{k}$

$r_u \times r_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4\sin u & 4\cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = (4\cos u - 0)\hat{\imath} - (-4\sin u - 0)\hat{\jmath} + 0\hat{k}$

$4\cos u\,\hat{\imath} + 4\sin u\,\hat{\jmath}$

$\|r_u \times r_v\| = \sqrt{16\cos^2 u + 16\sin^2 u} = 4$

$\int_0^2\int_0^\pi 4\,du\,dv = \int_0^2 4\pi\,dv = \boxed{8\pi}$