MTH 277, Quiz #13, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Write an equation for the arc length for $\vec{r}(t) = t\hat{\imath} + \frac{1}{9}t^3\hat{\jmath} + \frac{1}{4}t^2\hat{k}$ on the interval $[0,2]$. Evaluate it numerically.

$r'(t) = 1\hat{i} + \frac{1}{3}t^2\hat{j} + \frac{1}{2}t\hat{k}$

$\int_0^2 \sqrt{1 + \frac{t^4}{9} + \frac{t^2}{4}}\,dt \approx \boxed{2.54}$

2. Find the curvature of the curve $\vec{r}(t) = e^t\hat{\imath} + 4t\hat{\jmath}$ at the point $t = 1$. Use that information to find the radius of curvature at the same point.

$r'(t) = e^t\hat{i} + 4\hat{j}$

$r''(t) = e^t\hat{i} + 0\hat{j}$

$r' \times r'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ e^t & 4 & 0 \\ e^t & 0 & 0 \end{vmatrix} =$

$(0-0)\hat{i} - (0-0)\hat{j} + (0-4e^t)\hat{k}$

$\|r' \times r''\| = 4e^t$

$\|r'\| = \sqrt{e^{2t} + 16}$

$K = \frac{4e^t}{(e^{2t}+16)^{3/2}}$ $\quad K(1) = \frac{4e}{(e^2+16)^{3/2}} \approx .0961248$ $\quad R \approx \frac{1}{K} = 10.403...$

3. Find the curvature of $y = x^{2/3}$ at the point $x = \frac{1}{8}$.

$y' = \frac{2}{3}x^{-1/3}$

$y'' = -\frac{2}{9}x^{-4/3}$ $\quad |y''| = \frac{2}{9}x^{-4/3}$

$K = \frac{\frac{2}{9}x^{-4/3}}{\left(\sqrt{1 + \left(\frac{2}{3}x^{-1/3}\right)^2}\right)^3} = \frac{\frac{2}{9}x^{-4/3}}{\left(\sqrt{1 + \frac{4x^{-2/3}}{9}}\right)^3}$

$K(\frac{1}{8}) = \frac{\frac{2}{9}(16)}{\left(\frac{25}{9}\right)^{3/2}} = \frac{96}{125} = .768$